BROWNIAN ALTERNATOR

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Abstract

The two-resonator device and the pillar-array version thereof described in a previous manuscript are the subject of a more precise study. The physical interpretation of the device operation is changed for a more appropriate one. The hypothesis on which the pillar-array version is based is revisited and clearly formulated. The pillar-array device is considered under the aspect of an electric generator. This study leads to a new design for this generator. Another resonator is added in order to increase vibrations. The feasibility of an experimental device is addressed. Instructions for the fabrication of a prototype are given.

1. Introduction

In 2017 in [1], we were mostly focused on thermodynamic considerations; here, we envisage the devices described in Figs. 4 and 7a in [1] only in terms of energy production. The first device is made up of a nanopillar subjected to Brownian motion and coupled with a much bigger pillar, the Resonator (Fig. 1). In the second device, an array of nanopillars is used instead of a single nanopillar, and the Resonator has no longer the shape of a pillar; it is a piezoelectric thin film with two electrodes (Fig. 2). This last device is referred to as "the Brownian alternator" because, from the Brownian motions of nanopillars, it produces a permanent alternating voltage, like an alternating-current generator.

2. Single-Nanopillar Device

In this paper, the nanopillar and the Resonator of the device are described as coupled oscillators, either in a free-standing system (Fig. 3) or on a support (Fig. 4). The free-standing system is first studied in terms of the theory of coupled oscillators, which can be found in textbooks [2–5] on mechanics and radioelectricity (coupled oscillating circuits), under an assumption that there is no dissipation in the two oscillators. The nanopillar is characterized by displacement x_1, mass m, and spring constant k_1. The Resonator is characterized by displacement x_2, mass M, and spring constant k_2. The spring constant of the coupled oscillators is k. It can be found that $k = k_1k_2/k_1+k_2$; however, this value can be ignored in the calculations.

The nanopillar is subjected to Brownian motion, the time history of which looks like a sine wave of varying amplitude and phase yet constant frequency [6–9], with a mean amplitude a_1 given by:

$$a_1 = \left(\frac{k_B T}{k_1}\right)^{\frac{1}{2}}, \tag{1}$$

where k_B is the Boltzmann's constant and T is the absolute temperature.

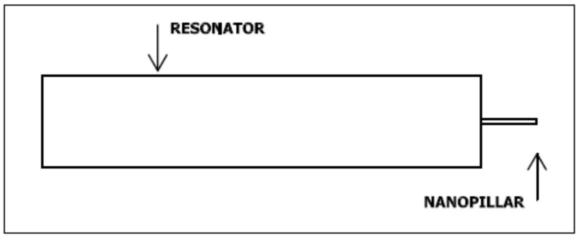

Fig. 1. Single-nanopillar device.

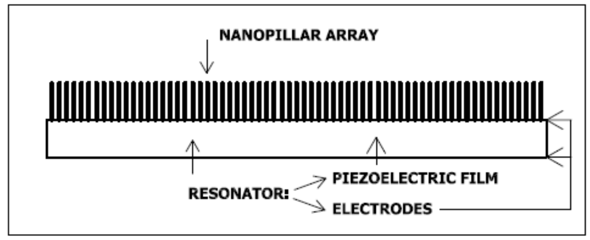

Fig. 2. Brownian alternator.

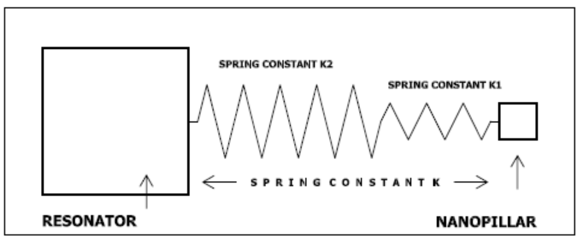

Fig. 3. Free-standing coupled oscillators.

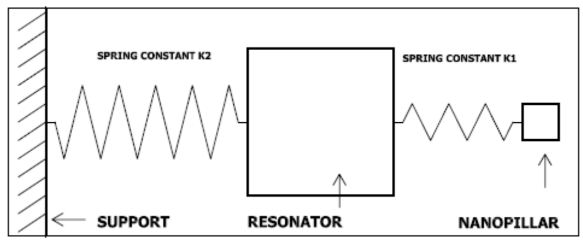

Fig. 4. Coupled oscillators on a support.

If the Resonator is at rest with $x_2=0$, the equation of motion of the nanopillar will be as follows (t is the time):

$$m\frac{d^2x_1}{dt^2} + kx_1 = 0, \text{ with the nanopillar resonant frequency } \omega_1 \text{ given by } \omega_1^2 = \frac{k}{m}, \tag{2; 3}$$

If the nanopillar is at rest with $x_1=0$, the equation of motion of the Resonator will be as follows:

$$M\frac{d^2x_2}{dt^2} + kx_2 = 0, \text{ with the Resonator's resonant frequency } \omega_2 \text{ given by } \omega_2^2 = \frac{k}{M}, \tag{4; 5}$$

The two equations of motion of the system of coupled oscillators are as follows:
for the nanopillar:

$$m\frac{d^2x_1}{dt^2} + kx_1 = kx_2, \text{ or using Eq. (3): } \frac{1}{\omega_1^2}\frac{d^2x_1}{dt^2} + x_1 = x_2, \tag{6; 7}$$

for the Resonator:

$$M\frac{d^2x_2}{dt^2} + kx_2 = kx_1, \text{ or using Eq. (5): } \frac{1}{\omega_2^2}\frac{d^2x_2}{dt^2} + x_2 = x_1, \tag{8; 9}$$

For the nanopillar Brownian motion, the model of a sinusoidal oscillation of amplitude a_1 is taken as follows:

$$x_1 = a_1\cos\omega t, \text{ and therefore: } \frac{d^2x_1}{dt^2} = -a_1\omega^2\cos\omega t, \tag{10; 11}$$

where ω is the angular frequency. Inserting the values of Eqs. (10) and (11) into Eq. (7), we obtain:

$$x_2 = \left(1 - \frac{\omega^2}{\omega_1^2}\right) a_1 \cos \omega t \text{ , and therefore: } \frac{d^2 x_2}{dt^2} = -\left(1 - \frac{\omega^2}{\omega_1^2}\right) a_1 \omega^2 \cos \omega t \,. \qquad (12;\,13)$$

Thus, the Resonator vibrates in phase or in phase opposition (depending on ω^2/ω_1^2) with respect to the nanopillar with an amplitude a_2 given by:

$$a_2 = \left(1 - \frac{\omega^2}{\omega_1^2}\right) a_1 \,, \qquad (14)$$

Inserting the values of Eqs. (12) and (13) into Eq. (9), we obtain:

$$\left[1 + \left(1 - \frac{\omega^2}{\omega_1^2}\right)\frac{\omega^2}{\omega_2^2} - \left(1 - \frac{\omega^2}{\omega_1^2}\right)\right] a_1 \cos \omega t = 0 \,, \qquad (15)$$

This equation implies that:

$$\omega^4 - \omega^2\left(\omega_1^2 + \omega_2^2\right) = 0 \,, \qquad (16)$$

which has two solutions ω' and ω'':

$$\omega'^2 = 0 \text{, and: } \omega''^2 = \omega_1^2 + \omega_2^2 \,, \qquad (17;\,18)$$

The nanopillar and the Resonator oscillate in phase opposition at frequency ω''. If both oscillators have the same resonant angular frequency ($\omega_2 = \omega_1$), that of the system is given by:

$$\omega'' = \omega_1 \sqrt{2} \,, \qquad (19)$$

From Eqs. (3) and (5), the Resonator vibration amplitude a_2 is as follows:

$$a_2 = \left(1 - \frac{\omega_1^2 + \omega_2^2}{\omega_1^2}\right) a_1 = -\frac{m}{M} a_1 \,, \qquad (20)$$

For the second system (coupled oscillators, both of resonant angular frequency ω_1', on a support), the equations of motion are as follows:
for the nanopillar:

$$m\frac{d^2 x_1}{dt^2} + k_1 x_1 = k_1 x_2 \,, \qquad (21)$$

for the Resonator:

$$M\frac{d^2 x_2}{dt^2} + \left(k_1 + k_2\right) x_2 = k_1 x_1 \,. \qquad (22)$$

Similar to the free-standing system, the calculations show that the Resonator vibration amplitude is exactly like that given by Eq. (20) with two solutions for the frequency of the coupled system:

$$\omega'^2 = \omega_1'^2\left(1 - \frac{m}{M}\right) \text{, (in-phase oscillations)} \qquad (23)$$

$$\omega''^2 = \omega_1'^2\left(1 + \frac{m}{M}\right) \text{, (phase-opposition oscillations)} \qquad (24)$$

with ω_1' given by:

$$\omega_1'^2 = \frac{k_1}{m} = \frac{k_1 + k_2}{M} \,, \qquad (25)$$

In the conclusion of Section 2, it is shown that, in terms of the model of coupled oscillators, the amplitude of the Resonator vibration is considerably smaller than that of the nanopillar. The vibration energy of the Resonator is also extremely low; therefore, it should be noted that there is no decrease in entropy in the single-pillar device studied in [1], contrary to what was written in that paper.

However, Eq. (20) allows us to catch sight of what should be done in order to get energy from Brownian motion. This is the subject of the next section.

3. Brownian Alternator

3.1. Design of the Brownian alternator

As noted in the Introduction, the “Brownian alternator” refers to a nanopillar-array device. The result of Eq. (20) indicates that the solution, to provide a large amplitude a_2 and then extract useful work from Brownian motion, is that the second oscillator should have the same mass as the first one.

For the Brownian alternator, this means that each nanopillar of the array should act on an extremely small part of the Resonator. This part should have the same mass as that of the nanopillar, so the shape of the Resonator has to be modified. Instead of a large pillar (see Fig. 6 in [1]), it takes the form of a thin film, the thickness of which is equal to the length of the nanopillars.

If this thin film were (in imagination) “cut” in small parts of the size of a nanopillar, this latter being only involved in the part facing itself, by gathering and “sticking” all these parts together, the uncut Resonator would vibrate as a whole at the nanopillar amplitude, with the essential condition being that the nanopillars vibrate in phase. This is the main condition for the device operation.

In the array, the nanopillars should be tightly packed by designing, for instance, hexagonal sections for the nanopillars with a narrow spacing between them.

In addition to the single-array design, the solution (Eqs. (21)–(25)) for the second system (coupled oscillators on a support) suggests another design (Fig. 5), where two devices are joined, each one acting as a support for the other one.

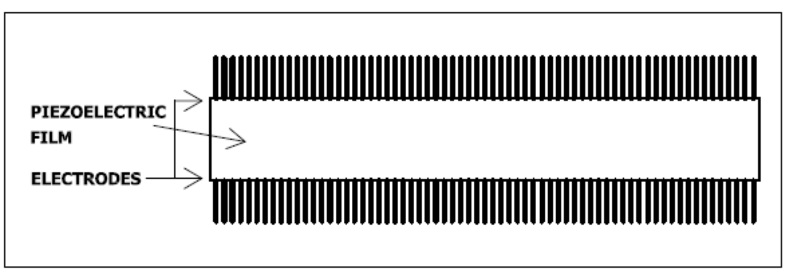

Fig. 5. Double-array Brownian alternator.

3.2. Operation principle

The following operation principle of the Brownian alternator is proposed: the Resonator has a much more regular vibration than that of the nanopillars subjected to the disorder, in amplitude and phase, of thermal agitation. However, the basis of coupling between oscillators is that they act on each other: each nanopillar of the array keeps the Resonator vibration going, and the Resonator acts on each nanopillar: if there is any out-of-phase nanopillar in the array, the coupling with the Resonator makes it come back to the in-phase state: the Resonator, because of vibrating as a whole, maintains the nanopillars of the array in phase (for starting the pillar in-phase motion see [1]).

To date, calculations and deductions have been made under the theoretical assumption of no dissipation in the Resonator. Now, in the actual case of dissipation, the Resonator amplitude can be smaller than that of the nanopillars; however, if dissipation is not too high, the two amplitudes are close to each other. The condition that the Brownian alternator also operates with low dissipation in the Resonator is still part of the previous proposition. This proposition explains how the condition of in-phase nanopillars of Section 3.1 could be fulfilled, and is nothing but a supposition. Therefore, the following hypothesis should be formulated.

3.3. Fundamental hypothesis

The working of the Brownian alternator relies on the following.

In the steady state of the system of coupled oscillators comprising an array of nanopillars subjected to Brownian motion and the underdamped Resonator, with the amplitude of the Resonator close to that of the nanopillars, we make the fundamental hypothesis that the Resonator maintains one single phase in the nanopillar Brownian motions.

These in-phase Brownian motions were referred to as “coherent Brownian motion” elsewhere [10].

3.4. Increased voltage with a resonant circuit

There are not many electric charges which appear at the electrodes of the piezoelectric film, because the vibration amplitude of the Resonator does not exceed that of the nanopillars. In order to get an increased amplitude, another resonator should be added to the Brownian alternator. This resonator is a resonant circuit connected to the electrodes, and the increased amplitude is that of a voltage, rather than of a mechanical vibration. The voltage inside the resonant circuit is considerably enhanced compared with that of the electrodes. The enhanced voltage makes the produced energy easier to use. Because of the size of the nanopillars, both the vibration of the piezoelectric film and the oscillation of the electric charges belong to the hyperfrequency range, and the resonant circuit would be rather a resonant cavity.

4. Experimental Device

The main part of the device is a piezoelectric film, which should have low losses and withstand hyperfrequency use in the 10 to 100 GHz range, depending on the nanopillar length. An array of nanopillars could be etched either in a thin film of any suitable material or directly on one of the two electrodes of the piezoelectric film [11–13]. Nanopillars with a hexagonal section, aspect ratio (length/diameter) of about 10, and a narrow spacing can be prepared by any technique providing smooth surfaces and accurate dimensions, for instance, by RIE or ICP [14]. The different thin layers constituting the device are as follows: the bottom electrode, the piezoelectric film, the top electrode, the nanopillar array. They could be deposited on a sacrificial layer, which can be easily removed to make the device free-standing, linked by electric connections to a support. The thickness of the layers should be determined by taking into account the wavelength of the frequency under consideration for each layer. The feasibility of the device does not present any insurmountable difficulty; it is within the reach of the present technology. Another issue is the hyperfrequency part of the device. It is a case for hyperfrequency physicists to carefully design the circuit to be connected to the electrodes of the Brownian alternator.

5. Discussion and Conclusion

Nanopillars driven by thermal agitation can be taken as sustained oscillators coupled with an underdamped oscillator, the Resonator. In this particular case, each nanopillar can be seen as having a negative mechanical resistance r_1, which cancels a small part of the positive mechanical resistance r_2 of the Resonator. If there are n nanopillars in the array, the effect of n negative r_1 is to cancel a large part or even the totality of r_2 [15]. In this last case, the Resonator amplitude $|a_2|$ equals the nanopillar amplitude $|a_1|$. It is evident that, in this system, the amplitude of the underdamped oscillator is dependent not only on r_2, but also on n and r_1.

The question is now arising, whether the n negative r_1 are able to cancel r_2, that is, which resistance of the Resonator the Brownian motions of the nanopillar array can cancel. This question is possibly related to the following one: how much time does it take, starting from amplitude zero, for thermal agitation to "refuel" nanopillar Brownian motion, that is, to reach the mean amplitude of Brownian motion? We suppose that this time is extremely short, possibly on the order of one period. These two questions could be the subject of a future paper.

In conclusion, we would like to underline that this paper on the Brownian alternator is far from being complete. It should be regarded as a contribution to the study of systems where nanosize elements (pillars) are in interaction with a macroscopic size component (Resonator). Within the framework of coupled oscillators we have shown that, although the single-pillar device does not supply any energy, useful work could be produced by the Brownian alternator, provided that the fundamental hypothesis is met. This hypothesis is worth testing with an experimental device. Current technology is able to produce high-quality piezoelectric thin films and high-aspect-ratio nanopillars, so that it would not be too difficult to build prototypes of the Brownian alternator.

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