# A TWO-DIMENSIONAL ELECTRON–HOLE SYSTEM IN TERMS OF THE CHERN–SIMONS THEORY: SLOWLY VARYING FIELD OPERATORS

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## Abstract

In the previous paper [1], the conduction and the valence electrons of the two-dimensional (2D) semiconductor layer subjected to the action of an external perpendicular magnetic field and interacting with 2D quantum point vortices have been described in terms of the Chern–Simons (C–S) theory. The C–S unitary transformation introducing the vector and scalar potentials generated by the quantum point vortices into the Hamiltonian and transforming the conduction and valence electrons into composite particles attaching them equal numbers of 2D quantum point vortices has been used. In the present paper, slowly varying envelope-type field operators are introduced. The initial Hamiltonian containing the periodic lattice potential and bare electron mass $m_0$ is transformed into the form with effective electron and hole masses, which determine the cyclotron frequencies of the Landau quantization in the presence of a C–S field.

## 1. Introduction

In [1], the Chern–Simons (C–S) theory developed by Jackiw and Pi [2–4] was applied to describe the system of a coplanar electron–hole gas subjected to the action of a strong perpendicular magnetic field interacting with two-dimensional (2D) quantum point vortices. They generate the C–S vector and scalar potentials and effective magnetic and electric fields. Even if these fields vanish in the mean-field approximations in the case of electrons and holes with equal densities, the quantum fluctuations generated by these fields will lead to new effects in the physics of the 2D systems. Unitary transformation operator $\hat{U}(\vec{r})$ introduced in [1] and bare $\hat{\psi}_0^+(r), \hat{\psi}_0(r)$ and dressed $\hat{\psi}^+(r), \hat{\psi}(r)$ electron field operators look as follows:

$$\begin{aligned} &\hat{U}(r) = e^{\frac{ie}{\hbar c}\hat{\omega}(\vec{r})};\ \hat{U}(\vec{r})\cdot U^+(\vec{r}) = 1\ \hat{\omega}^+(\vec{r}) = \omega(r) \\ &\hat{\psi}_0(\vec{r}) = \hat{U}(\vec{r})\hat{\psi}(\vec{r}),\ \hat{\psi}_0^+(\vec{r}) = \hat{\psi}^+(\vec{r})U^+(\vec{r}) \\ &\hat{\psi}(\vec{r}) = U^+(\vec{r})\hat{\psi}_0(\vec{r}),\ \psi^+(\vec{r}) = \psi_0^+(r)\hat{U}(\vec{r}) \end{aligned} \tag{1}$$

Singular phase operator $\hat{\omega}(\vec{r})$ and vector potential $\hat{\vec{a}}(\vec{r})$ were determined in [1] by formulas (1) and are repeated below:

$$\hat{\omega}(\vec{r}) = -\frac{\phi e}{\alpha}\int d^2\vec{r}\,'\theta(\vec{r}-\vec{r}\,')\hat{\rho}(r')$$

$$\hat{\vec{a}}(\vec{r}) = \vec{\nabla}_r\hat{\omega}(\vec{r}) = -\frac{\phi e}{\alpha}\int d^2\vec{r}\,'\vec{\nabla}_r\theta(\vec{r}-\vec{r}\,')\hat{\rho}(\vec{r}\,') \tag{2}$$

$$\hat{\rho}(\vec{r}) = \hat{\psi}_0^+(\vec{r})\hat{\psi}_0(\vec{r}) = \hat{\psi}^+(\vec{r})\hat{\psi}(\vec{r})$$

Here, $\phi$ is an integer number correlated with the filling factor $\upsilon$ in the case of the fractional quantum Hall effect (FQHE), namely, $\phi = \frac{1}{\upsilon}$; $\alpha$ is the fine structure constant $\alpha = \frac{1}{137}$ and the charge of the electron is $-e\left(e = |e|\right)$. The angle $\theta(\vec{r}-\vec{r}\,')$ is formed by the vector $\vec{r}-\vec{r}\,'$ with the $x$ axis on the plane of the layer and is determined analytically by the multivalued function

$$\theta(\vec{r}-\vec{r}\,') = \arctan\left(\frac{y-y'}{x-x'}\right),\ \theta(\vec{r}-\vec{r}\,') = \theta(\vec{r}\,'-\vec{r}) + \pi \tag{3}$$

The last relation in (3) represents the fact that the angle formed with the $x$ axis by vector $(\vec{r}\,'-\vec{r})$ differs by $\pi$ in comparison with vector $(\vec{r}-\vec{r}\,')$. It was shown in [2] that unitary transformation $\hat{U}(\vec{r})$ changes the statistical properties of transformed operators $\hat{\psi}^+(\vec{r})$ and $\hat{\psi}(\vec{r})$. This fact is well known in the conditions of the FQHE; it leads to the formation of composite particles predicted by Wilczek [5]. The composite particles look as bare electrons with an integer number $\phi$ of the attached 2D quantum point vortices.

If bare electron field operators $\hat{\psi}_0^+(\vec{r}),\ \hat{\psi}_0(r)$ satisfy the Fermi commutation relations expressed by formulas (4) in [1], then dressed electron field operators $\hat{\psi}^+(\vec{r})$ and $\hat{\psi}(\vec{r})$ obey the Fermi or to the Bose statistics as a function of integer number $\phi$, which must satisfy the condition

$$\pm e^{i\phi\pi} = 1 \tag{4}$$

An even integer $\phi = 0, 2, 4....$ corresponds to the Fermi statistics, whereas the composite particles with odd integer numbers $\phi = 1, 3, 5...$ of the vortices are bosons.

In [1], the equations of motion for the dressed electron field operators were deduced starting with initial Hamiltonian (25). It contains a periodic lattice potential; the effect of the external magnetic field and the Coulomb electron–electron interaction below the Coulomb electron–electron interaction will be dropped because the main task of the present paper is the introduction of slowly varying field operators instead of the full electron operators. We will determine the action of the C–S unitary transformation on the hole field operators instead of the valence electron ones. Another task is the excluding of the periodic lattice potential and the introduction of the effective electron and hole masses so as to simplify the description of the Landau quantization problem in the presence of the C–S vector potential. These questions will be solved in section 2 of the paper. Conclusions are made in section 3.

## 2. Equations of Motion for Dressed Electron Field Operators

The equations of motion described by formula (28) of [1] under stationary conditions neglecting the Coulomb electron–electron interaction look as follows:

$$\frac{\hbar^2}{2m_0}\left(-i\vec{\nabla}+\frac{e}{\hbar c}\vec{A}(\vec{r})+\frac{e}{\hbar c}\hat{\tilde{a}}(\vec{r})\right)^2\hat{\psi}(\vec{r})+V_{per}(\vec{r})\hat{\psi}(\vec{r})+\frac{e}{c}\frac{d\hat{\omega}(\vec{r})}{dt}\hat{\psi}(\vec{r})+\frac{e^2}{2\hbar c^2}\cdot i\hat{L}(\vec{r})\hat{\psi}(\vec{r})=E\hat{\psi}(\vec{r})$$

$$\frac{\hbar^2}{2m_0}\left(i\vec{\nabla}+\frac{e}{\hbar c}\vec{A}(\vec{r})+\frac{e}{\hbar c}\hat{\tilde{a}}(\vec{r})\right)^2\hat{\psi}^+(\vec{r})+V_{per}(\vec{r})\hat{\psi}^+(\vec{r})+\frac{e}{c}\hat{\psi}^+(\vec{r})\frac{d\hat{\omega}(\vec{r})}{dt}+ \quad (5)$$

$$+\frac{e^2}{2\hbar c^2}\hat{\psi}^+(\vec{r})\left(i\hat{L}(\vec{r})\right)^+=E\hat{\psi}^+(\vec{r})$$

Here, $\vec{A}(\vec{r})$ is the vector potential of the external magnetic field; the following relationships were used [1]:

$$\hat{\tilde{a}}(r)=\hat{\tilde{a}}(\vec{r})^+,\ \hat{\omega}(\vec{r})=\hat{\omega}^+(\vec{r});\ \left[\hat{\tilde{a}}(\vec{r}),\hat{\psi}(r)\right]=\left[\hat{a}(\vec{r}),\psi^+(r)\right]=0 \quad (6)$$

Operator $\frac{d\hat{\omega}(\vec{r})}{cdt}$ determines the scalar C–S potential, whereas the expression

$$i\hat{L}(\vec{r})=\frac{\hbar}{m_0}\left(\frac{\phi e}{a}\right)^2\int d^2\vec{r}\,'\left(\vec{\nabla}'\theta(\vec{r}-\vec{r}\,')\right)^2\hat{\rho}(\vec{r}\,')=\frac{\hbar}{m_0}\left(\frac{\phi e}{\alpha}\right)^2\int d^2\vec{r}\,'\frac{\hat{\rho}(\vec{r}\,')}{\left|\vec{r}-\vec{r}\,'\right|^2} \quad (7)$$

describes the energy created by the vortices. It is useful to remember that curl $\hat{\tilde{a}}(\vec{r})$ gives rise to an effective magnetic field [1] generated by the vortices.

Schrödinger equations (5) contain a periodic crystal lattice potential $V_{per}(\vec{r})$, which is invariant under the translation on integer lattice vector $\vec{R}$ enumerating the lattice nodes: $V_{per}(\vec{\rho}+\vec{R})=V_{per}(\vec{\rho})$. It was discussed in [6] that this property makes it possibleto represent an arbitrary space vector $\vec{r}$ in the form $\vec{r}=\vec{R}+\vec{\rho}$, where $\vec{\rho}$ is a small vector changing inside the unit lattice cell with volume $V_0=a_0^3$, where $a_0$ is the lattice constant. Due to this property, the electron Bloch wave functions can be represented as the products of periodic parts $U_{n,\vec{k}}(\vec{\rho})$ quickly changing inside the electron shells of the atoms situated in the unit cells and depending only on variable $\vec{\rho}$ and envelope wave functions $\varphi_{n,k}(\vec{R})$ slowly varying on variable $\vec{R}$. They almost do not depend on variable $\vec{\rho}$.

$$\psi_{nk}(\vec{r})=U_{nk}(\vec{\rho})\varphi_{n,\vec{k}}(R). \quad (8)$$

At the same time, the periodic parts quickly varying in space describe the semiconductor energy band structure quickly establishing in time, against the background of which the Landau quantization processes slowly varying in time take place. The dominance of the semiconductor energy band structure over other slowly varying processes makes it possible to introduce an adiabatic approximation and integrate over variable $\vec{\rho}$ as a method to exclude the quickly varying processes and concentrate the research on the physical processes slowly varying in space and time.

We are interested in these phenomena arising in a 2D electron–hole system and a 2D one-component electron gas.

The summation over the discrete lattice nodes of slowly varying envelope wave functions can be substituted by the integration over the surface area of the layer as follows:

$$\int d^2\vec{r} = \sum_{\vec{R}} \int_{S_0} d^2\vec{\rho} = \sum_{\vec{R}} a_0^2 \cdot \frac{1}{S_0} \int_{S_0} d^2\vec{\rho} = \int d^2\vec{R} \cdot \frac{1}{V_0} \int_{V_0} d^3\vec{\rho}$$
$$S_0 = a_0^2, V_0 = a_0^3, a_0^2 = d^2\vec{R}, \vec{r} = \vec{R} + \vec{\rho} \tag{9}$$

Here, the integration over surface area $S_0$ of the unit lattice cell was substituted by the integration over its volume $V_0$ because in reality 2D semiconductor structures have at least thickness $a_0$ of one atomic layer. Exactly following formula (11) of [6], vector potential $\vec{A}(\vec{r})$ of the external magnetic field and the required derivatives are shown below:

$$\vec{A}(\vec{r}) = \vec{A}(\vec{\rho}) + \vec{A}(\vec{R})$$
$$\frac{\partial}{\partial \vec{r}} \psi(\vec{\rho}, \vec{R}) = \frac{\partial}{\partial \vec{\rho}} \psi(\vec{\rho}, \vec{R}) + \frac{\partial}{\partial \vec{R}} \psi(\vec{\rho}, \vec{R}), \tag{10}$$
$$\hat{\vec{p}}_{\vec{r}} = \hat{\vec{p}}_{\rho} + \hat{\vec{p}}_{R},\ \vec{B}(\vec{\rho}) = rot_{\vec{\rho}} \vec{A}(\vec{\rho});\ \vec{B}(\vec{R}) = rot_{\vec{R}} \vec{A}(\vec{R})$$

It was mentioned in [6] that magnetic fields $\vec{B}(\vec{\rho})$ and $\vec{B}(\vec{R})$ are acting at different points of the real space: in the electron shell of the atoms situated inside the unit lattice cell giving rise to their Zeeman splitting effects and outside of it leading to the Landau quantization of the slowly moving electrons and holes.

In the two-band model of a semiconductor, using lattice variables $\vec{\rho}$ and $\vec{R}$, full electron field operators $\hat{\psi}(r)$ and $\hat{\psi}^+(\vec{r})$ looks as follows:

$$\hat{\psi}(\vec{r}) = \hat{\psi}(\vec{\rho}, \vec{R}) = U_c(\vec{\rho}) \hat{\varphi}_c(\vec{R}) + U_V(\vec{\rho}) \hat{\varphi}_V(\vec{R})$$
$$\hat{\psi}^+(\vec{r}) = \hat{\psi}^+(\vec{\rho}, \vec{R}) = U_c^*(\vec{\rho}) \hat{\varphi}_c^+(\vec{R}) + U_V^*(\vec{\rho}) \hat{\varphi}_V^+(\vec{R}) \tag{11}$$

where $U_c(\vec{\rho})$ and $U_V(\vec{\rho})$ are the periodic parts of the electron Bloch wave functions corresponding to the conduction and valence bands. They obey the normalization and orthogonality conditions:

$$\frac{1}{V_0} \int_{V_0} d^3\vec{\rho} \left| U_c(\vec{\rho}) \right|^2 = \frac{1}{V_0} \int_{V_0} d^3\vec{\rho} \left| U_V(\vec{\rho}) \right|^2 = 1$$
$$\frac{1}{V_0} \int_{V_0} d^3\vec{\rho} U_c^*(\vec{\rho}) U_V(\vec{\rho}) = 0$$
$$\frac{1}{V_0} \int_{V_0} d^3\vec{\rho} U_c^*(\vec{\rho}) \hat{\psi}(\vec{\rho}, \vec{R}) = \hat{\varphi}_c(\vec{R}) \tag{12}$$
$$\frac{1}{V_0} \int_{V_0} d^3\vec{\rho} U_V^*(\vec{\rho}) \hat{\psi}(\vec{\rho}, \vec{R}) = \hat{\varphi}_V(\vec{R})$$

Electron kinetic energy $\hat{K}(\vec{r})$ in variables $\vec{\rho}$ and $\vec{R}$ has the form

$$\hat{K}(\vec{r})=\frac{\hbar^2}{2m_0}\hat{\vec{\Pi}}(\vec{r})^2;\ \hat{K}^+(\vec{r})=\frac{\hbar^2}{2m_0}\hat{\vec{\Pi}}^+(\vec{r})^2$$

$$\hat{\vec{\Pi}}(\vec{r})=-i\vec{\nabla}_{\vec{r}}+\frac{e}{\hbar c}\vec{A}(\vec{r})+\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r})=-i\vec{\nabla}_{\vec{R}}-i\vec{\nabla}_{\vec{\rho}}+\frac{e}{\hbar c}\vec{A}(\vec{R})+$$

$$+\frac{e}{\hbar c}\vec{A}(\vec{\rho})+\frac{e}{\hbar c}\vec{a}(\vec{R})=\hat{\vec{\Pi}}(\vec{R})+\hat{\vec{\Pi}}(\vec{\rho}),$$

$$\hat{\vec{\Pi}}(\vec{R})=-i\vec{\nabla}_R+\frac{e}{\hbar c}\vec{A}(\vec{R})+\frac{e}{\hbar c}\hat{\vec{a}}(\vec{R}),\ \Pi^+(\vec{R})=i\vec{\nabla}_{\vec{R}}+\frac{e}{\hbar c}\vec{A}(\vec{R})+\frac{e}{\hbar c}\hat{\vec{a}}(\vec{R}) \tag{13}$$

$$\hat{\vec{\Pi}}(\vec{\rho})=-i\vec{\nabla}_{\vec{\rho}}+\frac{e}{\hbar c}\vec{A}(\vec{\rho}),\ \vec{A}^*(\vec{r})=\vec{A}(\vec{r}),\ \hat{\vec{a}}^+(\vec{R})=\hat{\vec{a}}(\vec{R}).$$

$$K(\vec{r})=\frac{\hbar}{2m_0}\left(\hat{\vec{\Pi}}(\vec{R})+\vec{\Pi}(\vec{\rho})\right)^2=\hat{K}(\vec{R})+\hat{K}(\vec{\rho})+\frac{\hbar^2}{m_0}\hat{\vec{\Pi}}(\vec{R})\cdot\hat{\vec{\Pi}}(\vec{\rho})$$

$$\hat{K}(\vec{R})=\frac{\hbar^2}{2m_0}\hat{\vec{\Pi}}(\vec{R})^2,\ \hat{K}(\vec{\rho})=\frac{\hbar^2}{2m_0}\hat{\vec{\Pi}}(\vec{\rho})^2$$

In these denotations, Schrödinger equations (5) take the form

$$E\hat{\psi}(\vec{\rho},\vec{R})=\Big[\hat{\mathsf{H}_0}(\vec{\rho})+\hat{K}(\vec{R})+\frac{\hbar^2}{m_0}\hat{\vec{\Pi}}(\vec{R})\cdot\hat{\vec{\Pi}}(\vec{\rho})+$$

$$+\frac{e}{c}\frac{d\hat{\omega}(\vec{R})}{dt}+\frac{e^2}{2\hbar c^2}i\hat{L}(\vec{R})\Big]\psi(\vec{\rho},\vec{R}).$$

$$E\psi^+(\vec{\rho},\vec{R})=\left[\hat{\mathsf{H}_0}^+(\vec{\rho})+\hat{K}^+(\vec{R})+\frac{\hbar^2}{m_0}\hat{\vec{\Pi}}(\vec{R})\cdot\hat{\vec{\Pi}}(\vec{\rho})\right]\psi^+(\vec{\rho},\vec{R})+ \tag{14}$$

$$+\frac{e}{c}\hat{\psi}^+(\vec{\rho},\vec{R})\frac{d\hat{\omega}(\vec{R})}{dt}+\frac{e^2}{2\hbar c^2}\psi^+(\vec{\rho},\vec{R})\left(i\hat{L}(\vec{R})\right)^+$$

In the deduction of these equations, the following properties were taken into account:

$$\vec{\nabla}_{\vec{R}}\cdot\vec{A}(\vec{R})=\vec{\nabla}_{\rho}\cdot\vec{A}(\vec{\rho})=0;\ \vec{\nabla}_{\vec{R}}\cdot\hat{\vec{a}}(\vec{R})=0,\ \left[\hat{\vec{a}}(\vec{r}),\hat{\psi}(r)\right]=0,\ \hat{\omega}^+(\vec{R})=\hat{\omega}(\vec{R})$$

(15)

Operator $\hat{\mathsf{H}_0}(\vec{\rho})$ consists of kinetic energy $\hat{K}(\vec{\rho})$ and periodic lattice potential $V_{per}(\vec{\rho})$ and determines the semiconductor band structure as follows:

$$\hat{\square}_0(\vec{\rho})=\hat{K}(\vec{\rho})+V_{per}(\vec{\rho})=\frac{\hbar^2}{2m_0}\left(-i\vec{\nabla}_{\vec{\rho}}+\frac{e}{\hbar c}A(\vec{\rho})\right)^2+V_{per}(\vec{\rho})$$

$$\hat{\square}_0(\vec{\rho})\cup_n(\vec{\rho})=E_nU_n(\rho),\ \square_0^+(\vec{\rho})U_n^*(\vec{\rho})=E_nU_n^*(\vec{\rho}),\ n=c,V,\ E_c-E_V=E_g^0. \tag{16}$$

The $E_c$ and $E_V$ values determine the bottom of the conduction band and the top of the valence band, respectively. The difference equals to energy band gap $E_g^0$. Multiplying equations (14) from the left-hand side by functions $U_c^*(\vec{\rho})$ and $U_V^*(\vec{\rho})$ and integrating them over volume $V_0$ of the lattice cell, we will transform them into reduced Schrödinger equations. The averaging with quickly varying wave functions $U_c(\rho)$ and $U_V(\vec{\rho})$ means the introduction of an adiabatic

approximation. In terms of this approximation, a semiconductor band structure is established. The remaining reduced Schrödinger equation describes only the slowly varying processes, such as the Landau quantization, that take place against the background of the already well established band structure. Relationships (16) lead to the following equalities:

$$\begin{aligned}
&\frac{1}{v_0}\int_{V_0} d^3\vec{\rho}\, U_c^*(\vec{\rho})\,\hat{\mathsf{H}}_0(\vec{\rho})\hat{\psi}(\vec{\rho},\vec{R}) = E_c\hat{\varphi}_c(\vec{R})\\
&\frac{1}{v_0}\int_{V_0} d^3\vec{\rho}\, U_V^*(\vec{\rho})\,\hat{\mathsf{H}}_0(\vec{\rho})\hat{\psi}(\vec{\rho},\vec{R}) = E_V\hat{\varphi}_V(\vec{R})\\
&\frac{1}{v_0}\int_{V_0} d^3\vec{\rho}\, U_V^*(\vec{\rho})\hat{\vec{\Pi}}(\vec{\rho})\hat{\psi}(\vec{\rho},\vec{R}) = \vec{\pi}_{c,V}\cdot\hat{\varphi}_V(\vec{R})\\
&\frac{1}{v_0}\int_{V_0} d^3\vec{\rho}\, U_V^*(\vec{\rho})\hat{\vec{\Pi}}(\vec{\rho})\hat{\psi}(\vec{\rho},\vec{R}) = \vec{\pi}_{V,c}\cdot\hat{\varphi}_c(\vec{R})
\end{aligned} \tag{17}$$

where matrix elements $\vec{\pi}_{n,m}$ were introduced:

$$\vec{\pi}_{c,V} = \frac{1}{v_0}\int_{V_0} d^3\vec{\rho}\, U_c^*(\vec{\rho})\hat{\vec{\Pi}}(\vec{\rho})U_V(\vec{\rho}) = \vec{\pi}_{V,c}^*;\ \vec{\pi}_{c,c} = \vec{\pi}_{V,V} = 0 \tag{18}$$

The Schrödinger equations for slowly varying field operators $\hat{\varphi}_c(\vec{R})$ and $\hat{\varphi}_V(\vec{R})$ in the adiabatic approximation are as follows:

$$\begin{aligned}
&E\hat{\varphi}_c(\vec{R}) = E_c\hat{\varphi}_c(\vec{R}) + \hat{K}(\vec{R})\hat{\varphi}_c(\vec{R}) + \frac{\hbar^2}{m_0}\hat{\vec{\Pi}}(\vec{R})\vec{\pi}_{cV}\hat{\varphi}_V(\vec{R}) +\\
&+\frac{e}{c}\frac{d\hat{\omega}(\vec{R})}{dt}\hat{\varphi}_c(R) + \frac{e^2}{2\hbar c^2}i\hat{L}(\vec{R})\hat{\varphi}_c(\vec{R}),\\
&E\hat{\varphi}_V(\vec{R}) = E_V\hat{\varphi}_V(\vec{R}) + \hat{K}(\vec{R})\hat{\varphi}_V(\vec{R}) + \frac{\hbar^2}{m_0}\hat{\vec{\Pi}}(\vec{R})\cdot\vec{\pi}_{Vc}\hat{\varphi}_c(\vec{R}) +\\
&+\frac{e}{c}\frac{d\hat{\omega}(\vec{R})}{dt} + \frac{e^2}{2\hbar c^2}i\hat{L}(\vec{R})\hat{\varphi}_V(\vec{R}).
\end{aligned} \tag{19}$$

Two equations (19) determine two slowly varying components $\hat{\varphi}_c(\vec{R})$ and $\hat{\varphi}_V(\vec{R})$ of field operator $\hat{\psi}(\vec{\rho},\vec{R})$. Their solutions are completely different at different values of energy $E$. In the range of $E = E_c + \varepsilon_e$ with $\varepsilon_e \ll E_c$, near the bottom of the conduction band, following the second equation (19), component $\hat{\varphi}_V(\vec{R})$ is small in comparison with component $\hat{\varphi}_c(\vec{R})$. Their relationship is as follows:

$$\hat{\varphi}_V(\vec{R}) = \frac{\hbar^2}{m_0 E_g^0}\hat{\vec{\Pi}}(\vec{R})\cdot\vec{\pi}_{V,c}\hat{\varphi}_c(\vec{R}), \tag{20}$$

whereas component $\hat{\varphi}_c(\vec{R})$ obeys the equation

$$\varepsilon_e\hat{\varphi}_c\left(R\right)=\left(\hat{K}\left(\vec{R}\right)+\frac{\hbar^4}{m_0^2E_g^0}\hat{\vec{\Pi}}\left(\vec{R}\right)\cdot\vec{\pi}_{cV}\cdot\hat{\vec{\Pi}}\left(\vec{R}\right)\cdot\vec{\pi}_{V,c}\right)\hat{\varphi}_c\left(\vec{R}\right)+$$
$$+\left(\frac{e}{c}\frac{d\hat{\omega}\left(\vec{R}\right)}{dt}+\frac{e^2}{2\hbar c^2}i\hat{L}\left(\vec{R}\right)\right)\hat{\varphi}_c\left(\vec{R}\right) \tag{21}$$

This procedure completely follows the method known as the envelope function approximation (EFA) developed by Winkler in his monograph [7]. After the simplification

$$\hat{\vec{\Pi}}\left(\vec{R}\right)\cdot\vec{\pi}_{cV}\cdot\hat{\vec{\Pi}}\left(\vec{R}\right)\vec{\pi}_{V,c}\approx\hat{\vec{\Pi}}\left(\vec{R}\right)^2\cdot\left|\vec{\pi}_{cV}\right|^2, \tag{22}$$

equation (21) takes the form

$$\varepsilon_e\hat{\varphi}_c\left(\vec{R}\right)=\left[\frac{\hbar^2}{2m_e}\hat{\vec{\Pi}}\left(\vec{R}\right)^2+\frac{e}{c}\frac{d\hat{\omega}\left(R\right)}{dt}+\frac{e^2}{2\hbar c^2}i\hat{L}\left(\vec{R}\right)\right]\hat{\varphi}_c\left(\vec{R}\right)$$
$$\frac{m_0}{m_e}=\left(1+\frac{2\hbar^2\left|\vec{\pi}_{c,V}\right|^2}{m_0E_g}\right) \tag{23}$$

If energy $E$ lies near the top of the valence band $E=E_V+\varepsilon_V$, with $\varepsilon_V \ll E_V$, then the dressed or composite electrons coincide with the valence electrons. Now, the first equation (19) with $E=E_V$ gives rise to the relationship

$$\hat{\varphi}_c\left(\vec{R}\right)=-\frac{\hbar^2}{m_0E_g^0}\cdot\hat{\vec{\Pi}}\left(\vec{R}\right)\cdot\vec{\pi}_{cV}\hat{\varphi}_V\left(\vec{R}\right), \tag{24}$$

whereas the second equation (19) looks as the reduced Schrödinger equation

$$\varepsilon_V\hat{\varphi}_V\left(\vec{R}\right)=\left[\frac{\hbar^2}{2m_V}\hat{\vec{\Pi}}\left(\vec{R}\right)^2+\frac{e}{c}\frac{d\omega\left(\vec{R}\right)}{dt}+\frac{e^2}{2\hbar c^2}i\hat{L}\left(\vec{R}\right)\right]\hat{\varphi}_V\left(\vec{R}\right) \tag{25}$$

with $m_V$ determined by the formula

$$\frac{m_0}{m_V}=\left(1-\frac{2\hbar^2\left|\vec{\pi}_{Vc}\right|^2}{m_0E_g^0}\right) \tag{26}$$

Our intention is to introduce the notion of the hole instead of the valence electron. For this purpose, the equation for Hermitian conjugate operator $\hat{\varphi}_V^+\left(R\right)$ is required. It can be obtained starting from the second equation (5) for field operator $\hat{\psi}^+\left(\vec{r}\right)$or directly from equation (25) as follows:

$$\varepsilon_V\hat{\varphi}_V^+\left(\vec{R}\right)=\frac{\hbar^2}{2m_V}\hat{\vec{\Pi}}^+\left(\vec{R}\right)^2\hat{\varphi}_V^+\left(\vec{R}\right)+\hat{\varphi}_V^+\left(\vec{R}\right)\left(\frac{e}{c}\frac{d\omega\left(\vec{R}\right)}{dt}+\frac{e^2}{2\hbar c^2}\left(i\hat{L}\left(R\right)\right)\right) \tag{27}$$

Here, the following relationships were used:

$$\left[\hat{\vec{a}}\left(r\right),\hat{\psi}\left(\vec{r}\right)\right]=0,\ \omega^+\left(\vec{R}\right)=\omega\left(\vec{R}\right);\ \left(i\hat{L}\left(\vec{R}\right)\right)^+=i\hat{L}\left(\vec{R}\right).$$

The hole has energy $\varepsilon_h$, effective mass $m_h$, and electrical charge $q_h$ of the opposite signs compared with that of the valence electron. The creation and annihilation operators of it are

expressed through the Hermitian conjugate of the valence electrons, respectively. They are as follows:

$$\hat{\varphi}_h^+\left(\vec{R}\right)=\varphi_V\left(\vec{R}\right),\ \hat{\varphi}_h\left(\vec{R}\right)=\varphi_V^+\left(\vec{R}\right)$$
$$\varepsilon_h=-\varepsilon_V,\ m_h=-m_V,\ \mathrm{q}_h=e \tag{28}$$

In these denotations, equation (27) looks as follows:

$$\varepsilon_h\hat{\varphi}_h\left(\vec{R}\right)=\frac{\hbar^2}{2m_h}\left(-i\vec{\nabla}_{\vec{R}}-\frac{e}{\hbar c}\vec{A}\left(\vec{R}\right)-\frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{R}\right)\right)^2\hat{\varphi}_h\left(\vec{R}\right)-$$
$$-\hat{\varphi}_h\left(\vec{R}\right)\left(\frac{e}{c}\frac{d\hat{\omega}\left(\vec{R}\right)}{dt}+\frac{e^2}{2\hbar c^2}i\hat{L}\left(\vec{R}\right)\right) \tag{29}$$

One can observe that the sign of the electric charge both inside kinetic momentum $\hbar\hat{\vec{\Pi}}\left(\vec{R}\right)$ and in other terms of equation (29) is opposite to the one of the conduction electron.

Thus, we have shown that the reduced Schrödinger equations for slowly varying electron and hole field operators $\hat{\varphi}_e\left(\vec{R}\right)=\hat{\varphi}_c\left(\vec{R}\right)$ and $\hat{\varphi}_h\left(\vec{R}\right)=\hat{\varphi}_V^+\left(\vec{R}\right)$, which after unitary transformation (1) form composite particles with effective masses $m_e$ and $m_h$ and with electrical charges $\mp e$ look as follows:

$$\varepsilon_e\hat{\varphi}_e\left(\vec{R}\right)=\frac{\hbar^2}{2m_e}\left(-i\vec{\nabla}_{\vec{R}}+\frac{e}{\hbar c}\vec{A}\left(\vec{R}\right)+\frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{R}\right)\right)^2\hat{\varphi}^+_{e\left(\vec{R}\right)}+\left(\frac{e}{c}\frac{d\hat{\omega}\left(\vec{R}\right)}{dt}+\frac{e^2}{2\hbar c^2}i\hat{L}\left(\vec{R}\right)\right)\hat{\varphi}_e\left(\vec{R}\right),$$
$$\varepsilon_h\hat{\varphi}_h\left(\vec{R}\right)=\frac{\hbar^2}{2m_h}\left(-i\vec{\nabla}_{R}-\frac{e}{\hbar c}\vec{A}\left(\vec{R}\right)-\frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{R}\right)\right)^2\hat{\varphi}_h\left(\vec{R}\right)-$$
$$-\hat{\varphi}_h\left(\vec{R}\right)\left(\frac{e}{c}\frac{d\hat{\omega}\left(\vec{R}\right)}{dt}+\frac{e^2}{2\hbar c^2}i\hat{L}\left(\vec{R}\right)\right). \tag{30}$$
$$\frac{m_0}{m_e}=\left(1+\frac{2\hbar^2\left|\vec{\pi}_{cV}\right|^2}{m_0E_g^0}\right);\ \frac{m_0}{m_h}=\left(\frac{2\hbar^2\left|\vec{\pi}_{cV}\right|^2}{m_0E_g^0}-1\right)$$
$$\hat{\varphi}_e\left(\vec{R}\right)=\hat{\varphi}_c\left(\vec{R}\right);\ \hat{\varphi}_h\left(\vec{R}\right)=\hat{\varphi}_V^+\left(\vec{R}\right)$$

Exactly the same equations can be obtained taking into account the Coulomb interactions between the electrons and the holes.

## 3. Conclusions

The paper is the continuation of the study published in the same issue [1], in which we have deduced the Schrödinger equations for the conduction and valence electrons of the 2D semiconductor layer subjected to the action of a perpendicular magnetic field and interacting with quantum point vortices in terms of the C–S theory. As new steps in this direction, the slowly varying electron and hole field operators in the adiabatic approximation have been introduced. To this end, the Schrödinger equations of [1] have been averaged using the periodic parts of the

electron Bloch wave functions and the reduced Schrödinger equation has been obtained. They are characterized by effective electron and hole masses and depend on the vector and scalar potentials created by the quantum point vortices in addition to the vector potential of the external magnetic field.

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