A TWO-DIMENSIONAL ELECTRON–HOLE SYSTEM UNDER CONDITIONS OF FRACTIONAL QUANTUM HALL EFFECTS

S. A. Moskalenko and V. A. Moskalenko

Institute of Applied Physics, str. Academiei 5, Chisinau, MD-2028 Republic of Moldova
E-mail: exciton@phys.asm.md

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Abstract

It has been shown that the Chern–Simons (C–S) gauge field created by quantum point vortices under conditions of fractional quantum Hall effects (FQHEs) leads to the formation of composite electrons and holes with equal integer numbers of quantum point vortices attached to each particle. The coherent superposition of the velocities of these vortices leads to the formation of the C–S vector potential, which depends on the difference between density operators $\hat{\rho}_e$ of the electrons and $\hat{\rho}_h$ of the holes. The C–S vector potential generates an effective magnetic field acting on the particles in addition to the external magnetic field. In the mean field approximation, when the average densities of electrons and holes coincide, the effective C–S magnetic and electric fields vanish and the Landau quantization of the composite particles with the bare electron and hole effective masses take place only under the action of the external magnetic field.

1. Introduction

Jackiw and Pi [1] have constructed a nonrelativistic field theory for a two-dimensional (2D) *N*-body system of point particles with C–S interactions. The C–S gauge field is created by the quantum point vortices located at each charged particle.

The C–S theory, which was developed by Jackiw and Pi [1] and widely used in the theory of fractional quantum Hall effects (FQHEs) [2–4], was applied to describe a 2D coplanar electron–hole system in a perpendicular magnetic field interacting with quantum point vortices [5, 6]. The motions of each electron and hole are accompanied by an integer number of quantum point vortices; these numbers are the same for electrons and for holes. The coherent summation of the angles formed with the in-plane x axis by the reference vectors between the positions of the given point and the points where the charged particles are situated, being weighted by the particle density operator, gives rise to phase operator $\hat{\omega}(\vec{r})$ of the C–S field. The field gradient $\vec{a}(r)=\vec{\nabla}\hat{\omega}(\vec{r})$ determines the vector-potential, whereas the time derivative $\frac{d\hat{\omega}}{cdt}$ generates the scalar potential. The C–S vector potential is a coherent superposition of the vortex velocities also weighted with the density operators of the point electrical charges. The quantum point vortices are localized only at the points where the charged particles do exist. Their vorticities are singular being different from zero only at these points. The C–S vector potential $\hat{\vec{a}}(\vec{r})$ generates an effective magnetic field, which is created by the electron and hole vortices and depends on the difference of their density operators $\hat{\rho}_e(\vec{r})$ and $\hat{\rho}_h(\vec{r})$ as follows: $\hat{\rho}(\vec{r})=\hat{\rho}_e(\vec{r})-\hat{\rho}_h(\vec{r})$. In our studies [5, 6] we have shown that the Landau quantization of the composite electrons and holes takes place with the effective masses of the bare electrons and holes mostly under the action of the external magnetic field, when the average values of density operators $\left\langle \hat\rho_e(r) \right\rangle$ and $\left\langle \hat\rho_h(\vec r) \right\rangle$ coincide $\left(\left\langle \hat\rho(\vec r) \right\rangle = 0\right)$.

In [5, 6], the C–S interaction was introduced in the Hamiltonian of the 2D layer containing the periodic lattice potential and the Coulomb electron–electron interaction in the presence of an external perpendicular magnetic field. A two-band model with the electrons filling the conduction and the valence band was considered. The exclusion of the periodic lattice potential and the possibility of introducing the motions of electrons and holes with effective masses in the presence of the C–S gauge field were demonstrated. In this study, on the basis of the previous findings, we are starting from the Hamiltonian in the electron–hole representation introducing the C–S gauge field with the unitary transformation. The paper is organized as follows. In section 2, the starting Hamiltonian in the electron–hole representation is introduced. In the third section, the C–S gauge field is introduced and the Schrödinger equations for the operators describing the composite particles are deduced. The fourth section contains the conclusions.

2. Hamiltonian of the Electron-Hole System

The Hamiltonian of the bare electrons and holes under the action of a strong perpendicular magnetic field, interacting between them by the Coulomb interaction, is expressed in terms of electron and hole field operators $\hat\psi_e^{0^+}(\vec r), \hat\psi_e^0(\vec r), \hat\psi_h^{0^+}(\vec r)$ and $\hat\psi_h^0(\vec r)$, and consist of the kinetic and Coulomb parts $\hat H^0 = \hat K^0 + \hat H^0_{Coul}$, where

$$\begin{aligned}\hat K^0 &= \frac{\hbar^2}{2m_e}\int d^2\vec r\,'\psi_e^{0+}(\vec r\,')\left(-i\vec\nabla' + \frac{e}{\hbar c}\vec A(\vec r\,')\right)^2 \hat\psi_e^0(\vec r\,') + \\ &+ \frac{\hbar^2}{2m_h}\int d^2\vec r\,'\hat\psi_h^{0+}(\vec r\,')\left(-i\vec\nabla' - \frac{e}{\hbar c}\vec A(\vec r\,')\right)^2 \hat\psi_h^0(\vec r\,').\end{aligned} \tag{1}$$

Here, $\vec A(\vec r)$ is the vector potential created by the external magnetic field perpendicular to the layer. It obeys to the condition $\vec\nabla \cdot \vec A(\vec r) = 0$ and, in the Landau gauge, has the form $\vec A(\vec r) = (-By, 0, 0),$, where B is the magnetic field strength. The Coulomb interaction Hamiltonian $\hat H^0_{Coul}$ between the bare electrons and holes has the form

$$\begin{aligned}&\hat H^0_{coul} = \frac{1}{2}\int d^2\vec r\,'\int d^2\vec r\,''V_{Coul}(\vec r\,' - \vec r\,'')\hat\psi_e^{0^+}(\vec r\,')\hat\psi_e^{0^+}(\vec r\,'')\hat\psi_e^0(\vec r\,'') \\ &\hat\psi_e^0(\vec r\,') + \frac{1}{2}\int d^2\vec r\,''V_{Coul}(\vec r\,' - \vec r\,'')\hat\psi_h^{0^+}(\vec r\,')\hat\psi_h^{0^+}(\vec r\,'')\hat\psi_h^0(\vec r\,'')\hat\psi_h^0(\vec r\,') - \\ &-\int d^2\vec r\,'\int d^2\vec r\,''V_{Coul}(\vec r\,' - \vec r\,'')\hat\psi_e^{0^+}(\vec r\,')\hat\psi_h^{0^+}(\vec r\,'')\hat\psi_h^0(\vec r\,'')\hat\psi_e^0(\vec r\,').\end{aligned} \tag{2}$$

The interaction potential $V_{Coul}(\vec r)$ in a 2D system can be represented as follows:

$$V_{Coul}\left(\vec{r}\right)=\sum_{\vec{Q}}V_{\vec{Q}}e^{i\vec{Q}\vec{r}},V_{\vec{Q}}=\frac{2\pi e^{2}}{\varepsilon_{0}S\left|\vec{Q}\right|}\tag{3}$$

Here, ε_0 is the dielectric constant and S is the layer surface area. Along with coefficient $V_{\vec{Q}}$, another expression containing the magnetic length l_0 will be used below, namely

$$W\left(\vec{Q}\right)=V_{\vec{Q}}e^{-\frac{Q^{2}l_{0}^{2}}{2}},l_{0}^{2}=\frac{\hbar c}{eB}.\tag{4}$$

The densities of the bare electron and hole numbers are introduced as follows:

$$\begin{gathered}\hat{\rho}_{e}^{0}\left(\vec{r}\right)=\hat{\psi}_{e}^{0^{+}}\left(\vec{r}\right)\hat{\psi}_{e}^{0}\left(\vec{r}\right);\ \hat{\rho}_{h}^{0}\left(\vec{r}\right)=\psi_{h}^{0^{+}}\left(\vec{r}\right)\psi_{h}^{0}\left(\vec{r}\right)\\ \hat{\rho}^{0}\left(\vec{r}\right)=\hat{\rho}_{e}^{0}\left(\vec{r}\right)-\hat{\rho}_{h}^{0}\left(\vec{r}\right);\ \hat{\rho}_{i}^{0^{+}}\left(\vec{r}\right)=\hat{\rho}_{i}^{0}\left(\vec{r}\right);\ \hat{\rho}_{i}^{0}\left(\vec{Q}=0\right)=\hat{N}_{i},\ i=e,h\\ \hat{\rho}_{i}^{0}\left(\vec{Q}\right)=\int d^{2}\vec{r}\,'\hat{\rho}_{i}^{0}\left(\vec{r}\right)e^{i\vec{Q}\vec{r}};\ \hat{\rho}_{i}^{0^{+}}\left(\vec{Q}\right)=\hat{\rho}_{i}^{0}\left(-\vec{Q}\right).\end{gathered}\tag{5}$$

The bare field operators in the absence of the quantum point vortices, which will be introduced latter, obey the Fermi commutation relations

$$\begin{gathered}\hat{\psi}_{i}^{0}\left(\vec{r}\right)\hat{\psi}_{j}^{0^{+}}\left(\vec{r}\,'\right)+\hat{\psi}_{j}^{0^{+}}\left(\vec{r}\,'\right)\hat{\psi}_{i}^{0}\left(\vec{r}\right)=\delta_{i,j}\delta^{2}\left(\vec{r}-\vec{r}\,'\right),\\ \hat{\psi}_{i}^{0}\left(\vec{r}\right)\hat{\psi}_{j}^{0}\left(\vec{r}\,'\right)+\hat{\psi}_{j}^{0}\left(\vec{r}\,'\right)\hat{\psi}_{i}^{0}\left(r\right)=0,\ i,j=e,h.\end{gathered}\tag{6}$$

The Coulomb interaction Hamiltonian $\hat{H}_{Coul}^{0}$ can be transcribed as follows:

$$\begin{gathered}\hat{H}_{Coul}^{0}=\frac{1}{2}\sum_{\vec{Q}}V_{\vec{Q}}\left[\hat{\rho}_{e}^{0}\left(\vec{Q}\right)\hat{\rho}_{e}\left(-\vec{Q}\right)-\hat{N}_{e}\right]+\\ +\frac{1}{2}\sum_{\vec{Q}}V_{\vec{Q}}\left[\hat{\rho}_{h}^{0}\left(\vec{Q}\right)\hat{\rho}_{h}^{0}\left(-\vec{Q}\right)-\hat{N}_{h}\right]-\sum_{\vec{Q}}V_{\vec{Q}}\hat{\rho}_{e}^{0}\left(\vec{Q}\right)\hat{\rho}_{h}^{0}\left(-\vec{Q}\right).\end{gathered}\tag{7}$$

The Schrödinger equations for operators $\hat{\psi}_{e}^{0}\left(\vec{r}\right)$ and $\hat{\psi}_{h}^{0}\left(\vec{r}\right)$ have the form

$$\begin{gathered}i\hbar\frac{d\hat{\psi}_{e}^{0}\left(\vec{r}\right)}{dt}=\left[\hat{\psi}_{e}^{0}\left(\vec{r}\right),\hat{H}^{0}\right]=\frac{\hbar^{2}}{2m_{e}}\left(-i\vec{\nabla}+\frac{e}{\hbar c}\vec{A}\left(\vec{r}\right)\right)^{2}\hat{\psi}_{e}^{0}\left(\vec{r}\right)+\\ +\int d^{2}\vec{r}\,'V_{Coul}\left(\vec{r}-\vec{r}\,'\right)\hat{\rho}^{0}\left(\vec{r}\,'\right)\hat{\psi}_{e}^{0}\left(\vec{r}\right);\\ i\hbar\frac{d\hat{\psi}_{h}^{0}\left(\vec{r}\right)}{dt}=\left[\hat{\psi}_{h}^{0}\left(\vec{r}\right),\hat{H}^{0}\right]=\frac{\hbar^{2}}{2m_{h}}\left(-i\vec{\nabla}-\frac{e}{\hbar c}\vec{A}\left(\vec{r}\right)\right)^{2}\hat{\psi}_{h}^{0}\left(\vec{r}\right)-\\ -\int d^{2}\vec{r}\,'V_{Coul}\left(\vec{r}-\vec{r}\,'\right)\hat{\rho}^{0}\left(\vec{r}\,'\right)\psi_{h}^{0}\left(\vec{r}\right);\\ i\hbar\frac{d\hat{\psi}_{e}^{0^{+}}\left(\vec{r}\right)}{dt}=-\frac{\hbar^{2}}{2m_{c}}\left(i\vec{\nabla}+\frac{e}{\hbar^{e}}\vec{A}\left(\vec{r}\right)\right)^{2}\hat{\psi}_{h}^{0^{+}}\left(\vec{r}\right)-\int d^{2}\vec{r}\,'V_{Coul}\left(\vec{r}-\vec{r}\,'\right)\\ \hat{\psi}_{e}^{0^{+}}\left(\vec{r}\right)\hat{\rho}^{0}\left(\vec{r}\,'\right);\ \hat{\rho}^{0}\left(\vec{r}\,'\right)=\hat{\rho}_{e}^{0}\left(\vec{r}\,'\right)-\hat{\rho}_{h}^{0}\left(\vec{r}\,'\right);\\ \frac{i\hbar d\hat{\psi}_{h}^{0^{+}}\left(\vec{r}\right)}{dt}=-\frac{\hbar^{2}}{2m_{h}}\left(i\vec{\nabla}-\frac{e}{\hbar c}\vec{A}\left(\vec{r}\right)\right)^{2}\hat{\psi}_{h}^{0^{+}}\left(\vec{r}\right)+\\ +\int d^{2}\vec{r}\,'V_{Coul}\left(\vec{r}-\vec{r}\,'\right)\hat{\psi}_{h}^{0^{+}}\left(\vec{r}\right)\hat{\rho}^{0}\left(\vec{r}\,'\right).\end{gathered}\tag{8}$$

On the basis of these equations of motion, time derivatives $\frac{d}{dt}\hat{\rho}_i^0\left(\vec{r}\right)$ were determined and the continuity equations were derived:

$$\begin{aligned}
&\frac{d}{dt}\hat{\rho}_i^0\left(\vec{r}\right)=\frac{d}{dt}\left(\hat{\psi}_i^{0^+}\left(\vec{r}\right)\hat{\psi}_i^0\left(\vec{r}\right)\right)=-\vec{\nabla}\cdot\vec{J}_i^0\left(\vec{r}\right),\ i=e,h.\\
&\vec{J}_e^0\left(\vec{r}\right)=\frac{\hbar}{2m_e i}\left(\hat{\psi}_e^{0^+}\left(\vec{r}\right)\vec{\nabla}\hat{\psi}_e^0\left(\vec{r}\right)-\vec{\nabla}\hat{\psi}_e^{0^+}\left(\vec{r}\right)\cdot\hat{\psi}_e^0\left(r\right)\right)+\frac{e}{m_e c}\vec{A}\left(\vec{r}\right)\hat{\rho}_e^0\left(\vec{r}\right)\\
&\vec{J}_h^0\left(\vec{r}\right)=\frac{\hbar}{2m_h i}\left(\hat{\psi}_h^{0^+}\left(\vec{r}\right)\vec{\nabla}\hat{\psi}_h^0\left(\vec{r}\right)-\nabla\hat{\psi}_h^{0^+}\left(\vec{r}\right)\cdot\hat{\psi}_h^0\left(\vec{r}\right)\right)-\frac{e}{m_h e}\vec{A}\left(\vec{r}\right)\hat{\rho}_h^0\left(\vec{r}\right).
\end{aligned} \tag{9}$$

To deduce formulas (8) and (9), the following commutation relations were used:

$$\begin{aligned}
&\left[\hat{\psi}_i^0\left(\vec{r}\right),\hat{\rho}_i^0\left(\vec{r}\,'\right)\right]=\delta^2\left(\vec{r}-\vec{r}\,'\right)\hat{\psi}_i^0\left(\vec{r}\,'\right)\\
&\left[\hat{\psi}_i^{0^+}\left(\vec{r}\right),\hat{\rho}_i^0\left(\vec{r}\,'\right)\right]=-\delta^2\left(\vec{r}-\vec{r}\,'\right)\hat{\psi}_i^{0^+}\left(\vec{r}\,'\right)\\
&\left[\hat{\rho}_i^0\left(\vec{r}\right),\hat{\rho}_j^0\left(\vec{r}\,'\right)\right]=0;\ \left[\hat{\rho}_i^0\left(\vec{r}\right),\hat{H}_{Coul}^0\right]=0;\ i,j=e,h
\end{aligned} \tag{10}$$

3. Chern–Simons Gauge Field

The effect of 2D quantum point vortices will be introduced in terms of the C–S theory using the unitary transformation from the bare electron and hole operators $\hat{\psi}_i^{0^+}\left(\vec{r}\right),\hat{\psi}_i^0\left(\vec{r}\right)$ to the new dressed electron and hole field operators $\hat{\psi}_i^+\left(\vec{r}\right),\hat{\psi}_i\left(\vec{r}\right)$ as follows:

$$\begin{aligned}
&\hat{\psi}_e^0\left(\vec{r}\right)=\hat{U}\left(\vec{r}\right)\hat{\psi}_e\left(\vec{r}\right),\hat{\psi}_e^{0^+}\left(\vec{r}\right)=\hat{\psi}_e^+\left(\vec{r}\right)\hat{U}^+\left(\vec{r}\right)\\
&\hat{\psi}_h^0\left(\vec{r}\right)=\hat{U}^+\left(\vec{r}\right)\hat{\psi}_h\left(\vec{r}\right),\psi_h^{0^+}\left(\vec{r}\right)=\hat{\psi}_h^+\left(\vec{r}\right)\hat{U}\left(\vec{r}\right)\\
&\hat{U}^+\left(\vec{r}\right)\cdot\hat{U}\left(\vec{r}\right)=1
\end{aligned} \tag{11}$$

They lead to the equalities of the bare and dressed density operators as follows:

$$\begin{aligned}
&\hat{\rho}_i\left(\vec{r}\right)=\hat{\psi}_i^+\left(\vec{r}\right)\hat{\psi}_i\left(\vec{r}\right)=\hat{\rho}_i^0\left(\vec{r}\right)=\hat{\psi}_I^{0^+}\left(\vec{r}\right)\psi_i^0\left(\vec{r}\right),\ i=e,h\\
&\hat{\rho}\left(\vec{r}\right)=\hat{\rho}_e\left(\vec{r}\right)-\hat{\rho}_h\left(\vec{r}\right)=\hat{\rho}^0\left(\vec{r}\right)=\hat{\rho}_e^0\left(\vec{r}\right)-\hat{\rho}_h^0\left(\vec{r}\right)\\
&\left[\hat{\rho}\left(r\right),\hat{\rho}\left(r\,'\right)\right]=0\ \hat{\rho}_i^+\left(\vec{r}\right)=\hat{\rho}_i\left(\vec{r}\right);\ \hat{\rho}^+\left(\vec{r}\right)=\hat{\rho}\left(\vec{r}\right)
\end{aligned} \tag{12}$$

Unitary transformation operator $\hat{U}\left(\vec{r}\right)$ was discussed in detail by Jackiw and Pi in [1]; we are completely following their explanations. In the two-component electron–hole system, we introduced it in the form

$$\begin{aligned}
&\hat{U}\left(\vec{r}\right)=e^{\frac{ie}{\hbar c}\hat{\omega}\left(\vec{r}\right)};\hat{\omega}\left(\vec{r}\right)=\frac{-\phi e}{\alpha}\int d^2\vec{r}\,'\theta\left(\vec{r}-\vec{r}\,'\right)\hat{\rho}\left(\vec{r}\,'\right),\\
&\hat{\rho}\left(\vec{r}\,'\right)=\hat{\rho}_e\left(\vec{r}\,'\right)-\hat{\rho}_h\left(\vec{r}\,'\right)=\hat{\rho}^0\left(\vec{r}\,'\right)=\hat{\rho}_e^0\left(\vec{r}\,'\right)-\vec{\rho}_h^0\left(\vec{r}\,'\right).
\end{aligned} \tag{13}$$

The phase operator and the unitary transformation contain factor ϕ with integer values $\phi=1,2,3...$ and a fine structure constant $\alpha=\frac{e^2}{\hbar c}=\frac{1}{137}$.

Function $\theta(\vec r-\vec r\,')$ is determined as the angle formed by the 2D vector $\vec r-\vec r\,'$ with the x axis lying in the plane of the layer. It is determined by the analytic formula

$$\Theta(\vec r-\vec r\,')=\arctan\left(\frac{y-y'}{x-x'}\right);\ \Theta(\vec r\,'-\vec r)=\Theta(\vec r-\vec r\,')+\pi \tag{14}$$

Function $\Theta(\vec r-\vec r\,')$ is multivalued being determined with the precision of 2π. Jackiw and Pi paid special attention to calculations of phase operator $\hat\omega(\vec r)$ and derivatives of it; these very instructive considerations will be reproduced later. In spite of these precautionary measures, unitary operator $\hat U(\vec r)$ and the dressed operators are single valued due to the integer values of the factor $\phi=1,2,3...$ Only these cases will be discussed.

The new dressed operators are introduced as follows:

$$\begin{aligned}&\hat\psi_e(\vec r)=\hat U^+(\vec r)\hat\psi_e^0(\vec r),\ \hat\psi_e^+(\vec r)=\hat\psi_e^{0^+}(\vec r)\hat U(\vec r),\\&\hat\psi_h(\vec r)=U(\vec r)\hat\psi_h^0(\vec r),\ \hat\psi_h^+(\vec r)=\hat\psi_h^{0^+}(\vec r)U^+(\vec r).\end{aligned} \tag{15}$$

As will be shown later, their statistics in general case differ from the Fermi statistics of the bare electron and hole operators $\hat\psi_i^{0^+}(\vec r),\hat\psi_i^0(\vec r)$. In the case of odd integer values $\phi=1,3,5...$ the dressed operators $\hat\psi_i^+(\vec r),\ \hat\psi_i(\vec r)$ obey the Bose statistics, whereas in the case of even integer values $\phi=0,2,4,....$ their statistics is Fermi. Nevertheless, the commutation relations of the all dressed operators with density operators $\hat\rho_i(\vec r)$ are the same as described by formulas (10).

It is evident from formulas (15) that, in the case of the electrons and the holes, the unitary transformation operations were effectuated using different Hermitian conjugated operators $\hat U(\vec r)$ and $\hat U^+(\vec r)$. The origin of this difference results from the definition of the hole field operator as the Hermitian conjugated field operator of the valence electron. Let us introduce the bare and dressed hole operators in the form

$$\hat\psi_h^0(\vec r)=\psi_{ve}^{0^+}(\vec r),\ \hat\psi_h(\vec r)=\psi_{ve}^+(\vec r) \tag{16}$$

where $\hat\psi_{ve}^0(r)$ and $\hat\psi_{ve}^{0^+}$ are the valence parts of the full bare electron operators $\hat\psi_e^0(r)$ and $\hat\psi_e^{0^+}(\vec r)$. We will take into account the relation between the $\hat\psi_{ve}^0(\vec r)$ and $\hat\psi_{ve}(r)$ as for the electron operators in the form $\hat\psi_{ve}^+(\vec r)=\hat\psi_{ve}^{0^+}(\vec r)\hat U(\vec r)$; in this case, we will arrive to the necessity to use the form $\hat\psi_h(\vec r)=\hat\psi_h^0(\vec r)\hat U(\vec r)$ instead of the form $\hat\psi_h(\vec r)=\hat U(r)\hat\psi_h^0(\vec r)$ accepted in the basic variant (11) and (15). In fact, the difference between them is not so significant because operators $\hat U(\vec r)$ and $\hat\psi_h^0(\vec r)$ can be transposed with the precision of the numerical phase factor derived below. In both versions, equality $\hat\rho_h(\vec r)=\hat\rho_h^0(\vec r)$ can be proved exactly on the basis of the relations derived below. Taking into account commutation relations (10) of the field operators with the density operators, one can obtain the transposition relations

$$\begin{aligned}
&\hat{\psi}_e\left(\vec{r}\right)e^{\pm\frac{ie}{\hbar c}\hat{\omega}\left(\vec{r}\,'\right)}=e^{\mp i\phi\theta\left(\vec{r}\,'-\vec{r}\right)}e^{\pm\frac{ie}{\hbar c}\hat{\omega}\left(\vec{r}\,'\right)}\hat{\psi}_e\left(\vec{r}\right),\\
&\hat{\psi}_e^+\left(\vec{r}\right)e^{\pm\frac{ie}{\hbar c}\hat{\omega}\left(\vec{r}\,'\right)}=e^{\pm i\phi\theta\left(\vec{r}\,'-\vec{r}\right)}e^{\pm\frac{ie}{\hbar c}\hat{\omega}\left(\vec{r}\,'\right)}\hat{\psi}_e^+\left(\vec{r}\right),\\
&\hat{\psi}_h\left(\vec{r}\right)e^{\pm\frac{ie}{\hbar c}\hat{\omega}\left(\vec{r}\,'\right)}=e^{\pm i\phi\theta\left(\vec{r}\,'-\vec{r}\right)}e^{\pm\frac{ie}{\hbar c}\hat{\omega}\left(\vec{r}\,'\right)}\hat{\psi}_h\left(\vec{r}\right),\\
&\psi_h^+\left(\vec{r}\right)e^{\pm\frac{ie}{\hbar c}\hat{\omega}\left(\vec{r}\,'\right)}=e^{\mp i\phi\theta\left(\vec{r}\,'-\vec{r}\right)}e^{\pm\frac{ie}{\hbar c}\hat{\omega}\left(\vec{r}\,'\right)}\hat{\psi}_h^+\left(\vec{r}\right).
\end{aligned}\tag{17}$$

The same relations are true in the case of the bare field operators, because the phase operator $\hat{\omega}\left(\vec{r}\right)$ can be expressed in equal manner through the bare or dressed density operators $\hat{\rho}\left(\vec{r}\right)=\hat{\rho}^0\left(\vec{r}\right)$, and commutation relations (10) are also the same in the both cases. Unitary transformations (11) and (15) guarantee the symmetrical description on equal footing of the electrons and the holes in the two-component system.

To deduce the Schrödinger equations for dressed field operators $\hat{\psi}_e\left(\vec{r}\right)$ and $\hat{\psi}_h\left(\vec{r}\right)$, the time derivatives of the unitary operators are required:

$$\begin{aligned}
&i\hbar\frac{d}{dt}\hat{\psi}_e\left(\vec{r}\right)=i\hbar\frac{d}{dt}\left(\hat{U}^+\left(r\right)\hat{\psi}_e^0\left(\vec{r}\right)\right)=\frac{i\hbar d\hat{U}^+\left(\vec{r}\right)}{dt}\cdot\hat{\psi}_e^0\left(\vec{r}\right)+\hat{U}^+\left(\vec{r}\right)\frac{i\hbar d\hat{\psi}_e^0\left(r\right)}{dt}\\
&i\hbar\frac{d}{dt}\hat{\psi}_h\left(\vec{r}\right)=i\hbar\frac{d}{dt}\left(\hat{U}\left(\vec{r}\right)\hat{\psi}_h^0\left(\vec{r}\right)\right)=\frac{i\hbar d\hat{U}\left(\vec{r}\right)}{dt}\cdot\psi_h^0\left(\vec{r}\right)+\hat{U}\left(\vec{r}\right)i\hbar\frac{d}{dt}\hat{\psi}_h^0\left(\vec{r}\right)
\end{aligned}\tag{18}$$

Jackiw and Pi [1] emphasized that it is necessary to take into account that the operator $\frac{d\hat{\omega}\left(r\right)}{dt}$ does not commute with operator $\hat{\omega}\left(\vec{r}\right)$. Their commutator $\hat{L}\left(\vec{r}\right)$ will be calculated later; however, the properties of it listed below

$$\begin{aligned}
&\left[\frac{d\hat{\omega}\left(\vec{r}\right)}{dt},\hat{\omega}\left(\vec{r}\right)\right]=-i\cdot\hat{L}\left(\vec{r}\right),\ \mathrm{L}^+\left(r\right)=\hat{L}\left(\vec{r}\right);\ \left[\hat{L}\left(\vec{r}\right),\hat{\rho}\left(\vec{r}\,'\right)\right]=0\\
&\left[\hat{L}\left(r\right),\hat{\omega}\left(\vec{r}\,'\right)\right]=0
\end{aligned}\tag{19}$$

will be used to determine the time derivates of the unitary transformation operators as follows [4, 5]:

$$\begin{aligned}
&\frac{d}{dt}\hat{U}(\vec{r})=\frac{d}{dt}e^{\frac{ie}{\hbar c}\hat{\omega}(\vec{r})}=[\frac{ie}{\hbar c}\frac{d\hat{\omega}(\vec{r})}{dt}+\frac{i\hat{L}(\vec{r})}{2}(\frac{ie}{\hbar c})^2]\hat{U}(\vec{r})=\hat{U}(\vec{r})[\frac{ie}{\hbar c}\frac{d\hat{\omega}(\vec{r})}{dt}-\frac{i\hat{L}(\vec{r})}{2}(\frac{ie}{\hbar c})^2];\\
&\frac{d}{dt}\hat{U}^+(\vec{r})=\frac{d}{dt}e^{-\frac{ie}{\hbar c}\hat{\omega}(\vec{r})}=[-\frac{ie}{\hbar c}\frac{d\hat{\omega}(\vec{r})}{dt}+\frac{i\hat{L}(\vec{r})}{2}(\frac{-ie}{\hbar c})^2]\hat{U}^+(\vec{r})=\hat{U}^+(\vec{r})[-\frac{ie}{\hbar c}\frac{d\hat{\omega}(\vec{r})}{dt}-\frac{i\hat{L}(\vec{r})}{2}(\frac{-ie}{\hbar c})^2].
\end{aligned}\tag{20}$$

They lead to the required derivatives

$$\begin{aligned}
&i\hbar\frac{d\hat{U}\left(\vec{r}\right)}{dt}=\left[-\frac{e}{c}\frac{d\hat{\omega}\left(\vec{r}\right)}{dt}+\frac{e^2}{2\hbar c^2}\hat{L}\left(\vec{r}\right)\right]\hat{U}\left(\vec{r}\right),\\
&i\hbar\frac{dU^+\left(\vec{r}\right)}{dt}=\left[\frac{e}{c}\frac{d\hat{\omega}\left(\vec{r}\right)}{dt}+\frac{e^2}{2\hbar c^2}\hat{L}\left(\vec{r}\right)\right]U^+\left(\vec{r}\right).
\end{aligned}\tag{21}$$

The calculations of operator $\hat{L}\left(\vec{r}\right)$ is considerably simplified working with the bare field operators as follows:

$$\begin{gathered}
-i\hat{L}\left(\vec{r}\right)=\left[\frac{d\hat{\omega}\left(\vec{r}\right)}{dt},\hat{\omega}\left(\vec{r}\right)\right]=\left(\frac{\phi e}{\alpha}\right)^{2}\int d^{2}\vec{r}\,'\int d^{2}\vec{r}\,''\theta\left(\vec{r}-\vec{r}\,'\right).\\
\theta\left(\vec{r}-\vec{r}\,''\right)\left\{\left[\frac{d\hat{\rho}_{e}^{0}\left(\vec{r}\,'\right)}{dt},\hat{\rho}_{e}^{0}\left(\vec{r}\,''\right)\right]+\left[\frac{d\hat{\rho}_{h}^{0}\left(\vec{r}\,'\right)}{dt},\hat{\rho}_{h}^{0}\left(\vec{r}\,''\right)\right]\right\}=\\
=-\left(\frac{\phi e}{\alpha}\right)^{2}\int d^{2}\vec{r}\,'\int d^{2}\vec{r}\,''\theta\left(\vec{r}-\vec{r}\,'\right)\theta\left(\vec{r}-\vec{r}\,''\right)\left\{\left[\vec{\nabla}'\vec{J}_{e}^{0}\left(\vec{r}\,'\right),\hat{\rho}_{e}^{0}\left(\vec{r}\,''\right)\right]+\left[\vec{\nabla}'\vec{J}_{h}^{0}\left(\vec{r}\,'\right),\hat{\rho}_{h}^{0}\left(\vec{r}\,''\right)\right]\right\}=\\
=\left(\frac{\phi e}{\alpha}\right)^{2}\int d^{2}\vec{r}\,'\int d^{2}\vec{r}\,''\vec{\nabla}'\theta\left(\vec{r}-\vec{r}\,'\right)\theta\left(\vec{r}-\vec{r}\,''\right)\times\left\{\left[\vec{J}_{e}^{0}\left(\vec{r}\,'\right),\hat{\rho}_{e}^{0}\left(\vec{r}\,''\right)\right]+\left[\vec{J}_{h}^{0}\left(\vec{r}\,'\right),\hat{\rho}_{h}^{0}\left(\vec{r}\,''\right)\right]\right\}.
\end{gathered}\tag{22}$$

Taking into account expressions (9) for current densities $\vec{J}_{e}^{0}\left(r'\right)$ and $\vec{J}_{h}^{0}\left(r'\right)$ and commutativity $\left[\hat{\rho}_{c}^{0}\left(r'\right),\rho_{i}^{0}\left(\vec{r}\,''\right)\right]=0$, expression (22) can be transcribed as follows:

$$\begin{gathered}
\hat{L}\left(\vec{r}\right)=\left(\frac{\phi e}{\alpha}\right)^{2}\frac{\hbar}{2m_{e}}\hat{M}_{e}\left(\vec{r}\right)+\left(\frac{\phi e}{\alpha}\right)^{2}\frac{\hbar}{2m_{h}}\hat{M}_{h}\left(\vec{r}\right),\\
\hat{M}_{i}\left(\vec{r}\right)=\int d^{2}\vec{r}\,'\int d^{2}\vec{r}\,''\vec{\nabla}'\theta\left(\vec{r}-\vec{r}\,'\right)\theta\left(\vec{r}-\vec{r}\,''\right)\left\{\left[\hat{\psi}_{i}^{0^{+}}\left(\vec{r}\,'\right)\vec{\nabla}'\hat{\psi}_{i}^{0}\left(\vec{r}\,'\right),\hat{\rho}_{i}^{0}\left(\vec{r}\,''\right)\right]-\right.\\
\left.-\left[\vec{\nabla}'\hat{\psi}_{i}^{0^{+}}\left(\vec{r}\,'\right)\hat{\psi}_{i}^{0}\left(\vec{r}\,'\right),\hat{\rho}_{i}^{0}\left(\vec{r}\,''\right)\right]\right\},\ i=e,h.
\end{gathered}\tag{23}$$

Using commutation relations (10), one can write

$$\begin{gathered}
\left[\hat{\psi}_{i}^{0^{+}}\left(\vec{r}\,'\right)\vec{\nabla}'\hat{\psi}_{i}^{0}\left(\vec{r}\,'\right),\hat{\rho}_{i}^{0}\left(\vec{r}\,''\right)\right]=\hat{\psi}_{i}^{0^{+}}\left(\vec{r}\,'\right)\vec{\nabla}'\left(\delta^{2}\left(\vec{r}\,'-\vec{r}\,''\right)\hat{\psi}_{i}^{0}\left(\vec{r}\,''\right)\right)-\\
-\delta^{2}\left(\vec{r}\,'-\vec{r}\,''\right)\psi_{i}^{0^{+}}\left(\vec{r}\,''\right)\vec{\nabla}'\hat{\psi}_{i}^{0}\left(\vec{r}\,'\right),\\
\left[\vec{\nabla}'\hat{\psi}_{i}^{0^{+}}\left(\vec{r}\,'\right)\hat{\psi}_{i}^{0}\left(\vec{r}\,'\right),\hat{\rho}_{i}^{0}\left(\vec{r}\,''\right)\right]=-\vec{\nabla}'\left(\delta^{2}\left(\vec{r}\,'-\vec{r}\,''\right)\hat{\psi}_{i}^{0^{+}}\left(\vec{r}\,''\right)\right)\times\\
\times\psi_{i}^{0}\left(\vec{r}\,'\right)+\vec{\nabla}'\hat{\psi}_{i}^{0^{+}}\left(\vec{r}\,'\right)\delta^{2}\left(\vec{r}\,'-\vec{r}\,''\right)\hat{\psi}_{i}^{0}\left(\vec{r}\,''\right),\ i=e,h.
\end{gathered}\tag{24}$$

We can now calculate integrals $\hat{M}_{i}\left(\vec{r}\right)$ as follows:

$$
\begin{aligned}
&\hat{M}_i(\vec r)=-\int d^2\vec r\,'\vec\nabla'\theta(\vec r-\vec r\,')\theta(\vec r-\vec r\,')\hat\psi_i^{0^+}(\vec r\,')\vec\nabla'\psi_i^0(\vec r\,')-\\
&-\int d^2\vec r\,'\vec\nabla'\theta(\vec r-\vec r\,')\theta(\vec r-\vec r\,')\vec\nabla'\psi_i^{0^+}(\vec r\,')\psi_i^0(\vec r\,')+\\
&+\int d^2\vec r\,''\int d^2\vec r\,'\vec\nabla'\theta(\vec r-\vec r\,')\theta(\vec r-\vec r\,'')\psi_i^{0^+}(\vec r\,')\vec\nabla'\left(\delta^2(\vec r\,'-\vec r\,'')\hat\psi_i^0(\vec r\,'')\right)+\\
&+\int d^2\vec r\,''\int d^2\vec r\,'\vec\nabla'\theta(\vec r-\vec r\,')\theta(\vec r-\vec r\,'')\vec\nabla'\left(\delta^2(\vec r\,'-\vec r\,'')\hat\psi_i^{0^+}(\vec r\,'')\right)\psi_i^0(\vec r\,')=\\
&=-\int d^2\vec r\,'\vec\nabla'\theta(\vec r-\vec r\,')\theta(\vec r-\vec r\,')\vec\nabla'\hat\rho_i^0(\vec r\,')+\int d^2\vec r\,''\theta(\vec r-\vec r\,'')\cdot\int d^2\vec r\,'\cdot\\
&\cdot\vec\nabla'\theta(\vec r-\vec r\,')\left[\hat\psi_i^{0^+}(\vec r\,')\vec\nabla'\left(\delta^{(2)}(\vec r\,'-\vec r\,'')\psi_i^0(\vec r\,'')\right)+\vec\nabla'\left(\delta^{(2)}(\vec r\,'-\vec r\,'')\cdot\psi_i^{0^+}(\vec r\,'')\right)\psi_i^0(\vec r\,')\right]=\\
&=\int d^2\vec r\,'\vec\nabla'\left(\vec\nabla'\theta(\vec r-\vec r\,')\theta(\vec r-\vec r\,')\right)\cdot\hat\rho_i^0(\vec r)+\int d^2\vec r\,''\theta(\vec r-\vec r\,'')\\
&\left[\int d^2\vec r\,'\vec\nabla'\theta(\vec r-\vec r\,')\hat\psi_i^{0^+}(\vec r\,')\vec\nabla'\left(\delta^{(2)}(\vec r\,'-\vec r\,'')\cdot\hat\psi_i^0(\vec r\,'')\right)+\right.\\
&\left.+\int d^2\vec r\,'\vec\nabla'\theta(\vec r-\vec r\,')\vec\nabla'\left(\delta^2(\vec r\,'-\vec r\,'')\hat\psi_i^{0^+}(\vec r\,'')\right)\psi_i^0(\vec r\,')\right].
\end{aligned}
\tag{25}
$$

Here, we should take into account the equality $\Delta'\theta(\vec r-\vec r\,')=0$, which simplifies the next calculation:

$$
\begin{aligned}
&M_i(\vec r)=\int d^2\vec r\,'\left(\vec\nabla'\theta(\vec r-\vec r\,')\right)^2\hat\rho_i^0(\vec r\,')-\int d^2\vec r\,'\int d^2\vec r\,''\theta(\vec r-\vec r\,'')\vec\nabla'\theta(\vec r-\vec r\,')\cdot\\
&\cdot\vec\nabla'\hat\psi_i^0(\vec r\,')\hat\psi_i^0(\vec r\,'')\delta^2(\vec r\,'-\vec r\,'')-\int d^2\vec r\,'\int d^2\vec r\,''\theta(\vec r-\vec r\,'')\vec\nabla'\theta(\vec r-\vec r\,')\hat\psi_i^{0^+}(\vec r\,'')\cdot\\
&\cdot\vec\nabla'\hat\psi_i^0(\vec r\,')\delta^2(\vec r\,'-\vec r\,'')=2\int d^2\vec r\,'\left(\vec\nabla'\theta(\vec r-\vec r\,')\right)^2\hat\rho_i^0(\vec r\,'),\ i=e,h.
\end{aligned}
\tag{26}
$$

$$
\begin{gathered}
\hat L(\vec r)=\left(\frac{\phi e}{\alpha}\right)^2\hbar\left[\frac{1}{m_e}\int d^2\vec r\,'\left(\vec\nabla'\theta(\vec r-\vec r\,')\right)^2\hat\rho_e^0(\vec r\,')+\frac{1}{m_h}\int d^2\vec r\,'\left(\vec\nabla'\theta(\vec r-\vec r\,')\right)^2\hat\rho_h^0(\vec r\,')\right]\\
\left[\hat L(\vec r),\hat\omega(\vec r\,')\right]=0;\ \left[\hat L(\vec r),\hat\rho(\vec r\,')\right]=0;\ \hat L^+(r)=\hat L(\vec r).
\end{gathered}
\tag{27}
$$

To derive the Schrödinger equation for dressed field operators $\hat\psi_i(\vec r)$ following formulas (17), it is necessary to determine expressions $\hat U^+(\vec r)i\hbar\dfrac{d\hat\psi_e^0(\vec r)}{dt}$ and $\hat U(\vec r)i\hbar\dfrac{d\psi_h^0(\vec r)}{dt}$. To this end, the equalities [1] were used below:

$$
\begin{gathered}
\hat U^+(\vec r)(-i\vec\nabla+\frac{e}{\hbar c}\vec A(\vec r))^2\hat\psi_e^0(\vec r)=(-i\vec\nabla+\frac{e}{\hbar c}\vec A(\vec r)+\frac{e}{\hbar c}\vec a(\vec r))^2\hat\psi_e(\vec r),\\
\hat U(\vec r)(-i\vec\nabla-\frac{e}{\hbar c}\vec A(\vec r))^2\hat\psi_h^0(\vec r)=(-i\vec\nabla-\frac{e}{\hbar c}\vec A(\vec r)-\frac{e}{\hbar c}\vec a(\vec r))^2\hat\psi_h(\vec r),\\
\hat\psi_e(\vec r)=\hat U^+(\vec r)\hat\psi_e^0(\vec r);\quad \hat\psi_h(\vec r)=\hat U(\vec r)\hat\psi_h^0(\vec r).
\end{gathered}
\tag{28}
$$

Here, $\hat{\vec a}(\vec r)$ is the C–S vector-potential determined by the definition

$$
\hat{\vec a}(\vec r)=\vec\nabla\hat\omega(\vec r)=-\frac{\phi e}{\alpha}\int d^2\vec r\,'\vec\nabla\theta(\vec r-\vec r\,')\hat\rho(\vec r\,'). \tag{29}
$$

Jackiw and Pi [1] in their fundamental studies of the C–S theory paid attention to the expression of type (29) to discussing the possibility of moving the gradient with respect to $\vec r$ out of the integral on variable $\vec r\,'$.

They mentioned that, in the general case, operator $\vec{U}(\vec{r})$ is singular, because $\theta(\vec{r}-\vec{r}\,')$ is a multivalued function and the integration over 2D $\vec{r}\,'$ plane requires the cut in $\vec{r}\,'$ beginning in $\vec{r}$. However, in the nonrelativistic quantum mechanics, the particles are points and the matter density operator $\hat{\rho}(\vec{r})$ is localized at these points being a superposition of δ-functions. This fact plays a crucial role in the calculations involving the C–S gauge field. It makes it possible to interchange the integration and the differentiation in the definition (29) of the C–S vector-potential. Jackiw and Pi [1] emphasized that the presence of the density operator $\hat{\rho}(\vec{r})$ with δ-function eigenvalues in the integral leads to an exceptional situation, when the $\vec{r}$ gradient can be moved outside the integral with impunity. In [1], some peculiarities of the 2D space were enumerated, such as

$$\begin{gathered}
curl=\vec{\nabla}\times=\vec{e}_x\frac{\partial}{\partial y}-\vec{e}_y\frac{\partial}{\partial x};\ \vec{\nabla}=\vec{e}_x\frac{\partial}{\partial x}+\vec{e}_y\frac{\partial}{\partial y}\\
\vec{\nabla}\theta(\vec{r}-\vec{r}\,')=-\vec{\nabla}\times\ln|\vec{r}-\vec{r}\,'|;\ \vec{\nabla}\times\theta(\vec{r}-\vec{r}\,')=\vec{\nabla}\ln|\vec{r}-\vec{r}\,'|\\
\Delta\theta(\vec{r}-\vec{r}\,')=0;\ \Delta\ln|\vec{r}-\vec{r}\,'|=2\pi\delta^{(2)}(\vec{r}-\vec{r}\,')\qquad(30)\\
\vec{\nabla}\cdot\vec{a}(\vec{r})=0;\ curl\hat{\vec{a}}(\vec{r})=\vec{\nabla}\times\vec{a}(\vec{r})=\hat{b}(\vec{r})=\vec{\nabla}\times\frac{\phi e}{\alpha}\int d^2\vec{r}\,'\hat{\rho}(\vec{r}\,')\vec{\nabla}\times\ln|\vec{r}-\vec{r}\,'|=\\
=\frac{\phi e}{\alpha}\int d^2\vec{r}\,'\hat{\rho}(\vec{r}\,')\Delta\ln|\vec{r}-\vec{r}\,'|=\frac{2\pi\phi e}{\alpha}\hat{\rho}(\vec{r}).
\end{gathered}$$

The curl $\hat{\vec{a}}(\vec{r})$ determines the effective magnetic field strength $\hat{b}(\vec{r})$ created by the vortices. In the two-component electron–hole system it equals to

$$\hat{b}(\vec{r})=curl\hat{\vec{a}}(\vec{r})=\frac{2\pi\phi e}{\alpha}\hat{\rho}(\vec{r})=\frac{2\pi\phi e}{\alpha}\left(\hat{\rho}_e(\vec{r})-\hat{\rho}_h(\vec{r})\right)\tag{31}$$

In the mean field approximation, where the average values $\langle\hat{\rho}_e(\vec{r})\rangle$ and $\langle\hat{\rho}_h(\vec{r})\rangle$ coincide, the effective magnetic field strength $\langle\hat{b}(\vec{r})\rangle$ created by the quantum point vortices vanishes.

The gradient of the multivalued function $\theta(\vec{r})$ being considered alone, without density operator, gives rise to velocity field $\vec{V}(\vec{r})$ with singular vorticity $\Omega(\vec{r})$. In fact,

$$\begin{gathered}
\vec{V}(\vec{r})=\frac{-k}{2\pi}\vec{\nabla}\theta(\vec{r})=\frac{k}{2\pi}\vec{\nabla}\times\ln(\vec{r})=\frac{-k}{2\pi}\left(\frac{-\vec{e}_x y+\vec{e}_y x}{\vec{r}^{\,2}}\right)\\
\Omega(\vec{r})=curl\vec{V}(\vec{r})=\vec{\nabla}\times\vec{V}(\vec{r})=\frac{k}{2\pi}\Delta\ln\vec{r}=k\delta^{(2)}(\vec{r}).
\end{gathered}\tag{32}$$

The C–S vector potential $\hat{\vec{a}}(\vec{r})$ arises due to the summation of velocities $\vec{V}(\vec{r})$ created by all vortices attached to the electrons and holes at these points of the 2D space, where the densities of the charges $\hat{\rho}_i(\vec{r})$ are different from zero. The electrons and holes with the attached quantum point vortices form composite particles. They have been first introduced in physics by Wilczek [6], yet in a slightly different way. The contemporary interpretation of the composite particles structure was proposed by Read [3]. The statistical properties of the dressed field operators describing the composite electrons and holes will be discussed below. As mentioned earlier, the bare electron and hole field operators obey the Fermi statistics, whereas the dressed field operators will obey the Fermi or Bose statistics depending on the parity of the integer values of factor ϕ. To obey the Fermi or Bose statistics, the dressed field operators should satisfy the following requirements:

$$\begin{aligned}&\hat{\psi}_i\left(\vec{r}\right)\hat{\psi}_j^+\left(\vec{r}\,'\right)\pm\hat{\psi}_j^+\left(\vec{r}\,'\right)\hat{\psi}_i\left(\vec{r}\right)=\delta_{i,j}\delta^{(2)}\left(\vec{r}-\vec{r}\,'\right)\\&\hat{\psi}_i\left(\vec{r}\right)\hat{\psi}_j^+\left(\vec{r}\,'\right)\pm\hat{\psi}_j^+\left(\vec{r}\,'\right)\hat{\psi}_i\left(\vec{r}\right)=0;\ i,j=e,h\end{aligned}\tag{33}$$

Here, the upper and lower signs concern to Fermi or to Bose statistics, respectively. We will determine the integer values of factor ϕ, which are compatible with requirements (33) using the electron field operators $\hat{\psi}_e\left(r\right)=U^+\left(r\right)\hat{\psi}_e^0\left(\vec{r}\right)$ and $\hat{\psi}_e^+\left(\vec{r}\right)=\hat{\psi}_e^{0^+}\left(r\right)\hat{U}\left(\vec{r}\right)$ as a particular example. Substituting them into the first equation (33) and taking into account the transposition relations (17), we will find

$$\begin{aligned}&\hat{U}^+\left(\vec{r}\right)\hat{\psi}_e^0\left(\vec{r}\right)\hat{\psi}_e^{0^+}\left(\vec{r}\,'\right)\hat{U}\left(\vec{r}\,'\right)\pm\psi_e^{0^+}\left(\vec{r}\,'\right)\hat{U}\left(\vec{r}\,'\right)\hat{U}^+\left(\vec{r}\right)\hat{\psi}_e^0\left(\vec{r}\right)=\\&=e^{i\phi\cdot\theta(0)}\left[\hat{U}^+\left(\vec{r}\right)\hat{\psi}_e^0\left(\vec{r}\right)\hat{U}\left(\vec{r}\,'\right)\hat{\psi}_e^{0^+}\left(\vec{r}\,'\right)\pm\hat{U}\left(\vec{r}\,'\right)\hat{\psi}_e^{0^+}\left(\vec{r}\,'\right)\hat{U}^+\left(\vec{r}\right)\hat{\psi}_e^0\left(\vec{r}\right)\right]=\\&=e^{i\phi\theta(0)}\left[e^{-i\phi\theta(\vec{r}\,'-\vec{r})}\hat{U}^+\left(\vec{r}\right)\hat{U}\left(\vec{r}\,'\right)\hat{\psi}_e^0\left(\vec{r}\right)\hat{\psi}_e^{0^+}\left(\vec{r}\,'\right)\pm e^{-i\phi\theta(\vec{r}-\vec{r}\,')}\hat{U}\left(\vec{r}\,'\right)\hat{U}^+\left(\vec{r}\right)\hat{\psi}_e^{0^+}\left(\vec{r}\,'\right)\hat{\psi}_e^0\left(\vec{r}\right)\right]=\delta^2\left(\vec{r}-\vec{r}\,'\right)\end{aligned}$$

(34)

Due to the commutativity $\hat{U}\left(\vec{r}\,'\right)\hat{U}^+\left(r\right)=\hat{U}^+\left(\vec{r}\right)\hat{U}\left(\vec{r}\,'\right)$ and the relation $\theta\left(\vec{r}-\vec{r}\,'\right)=\theta\left(\vec{r}\,'-\vec{r}\right)-\pi$, requirement (34) can be transcribed as follows:

$$\begin{aligned}&\hat{\psi}_e\left(\vec{r}\right)\hat{\psi}_e^+\left(\vec{r}\,'\right)\pm\hat{\psi}_e^+\left(\vec{r}\,'\right)\hat{\psi}\left(\vec{r}\right)=e^{i\phi\left(\theta(0)-\theta(\vec{r}\,'-\vec{r})\right)}\hat{U}^+\left(\vec{r}\right)\hat{U}\left(\vec{r}\,'\right)\times\\&\times\left[\hat{\psi}_e^0\left(\vec{r}\right)\hat{\psi}_e^{0^+}\left(\vec{r}\,'\right)\pm e^{i\phi\pi}\hat{\psi}_e^{0^+}\left(\vec{r}\,'\right)\hat{\psi}_e^0\left(\vec{r}\right)\right]=\delta^{(2)}\left(\vec{r}-\vec{r}\,'\right)\end{aligned}\tag{35}$$

The requirement can be satisfied, if

$$\pm e^{i\phi\pi}=1,\ \cos\phi\pi=\pm1;\ \begin{bmatrix}F\\B\end{bmatrix}\tag{36}$$

The same conditions take place for the second equation (33) as well as for the hole field operators. It means that dressed field operators $\hat{\psi}_i\left(\vec{r}\right)$ with unitary transformations $\hat{U}\left(\vec{r}\right),\ \hat{U}^+\left(\vec{r}\right)$ and with even integer values $\phi=0,2,4...$ obey the Fermi statistics, whereas in the case of the odd integer values $\phi=1,3,5,...$, they obey the Bose statistics.

The composite particles composed of a bare electron or a hole with an odd integer number ϕ of attached quantum point vortices are bosons, whereas in the case of an even integer number ϕ of attached vortices, they are fermions.

Combining expressions (8), (18), (21), and (28), we will obtain the Schrödinger equations for the dressed field operators

$$
\begin{aligned}
& i\hbar \frac{d}{dt}\hat{\psi}_e(\vec{r}) = \frac{\hbar^2}{2m_e}[-i\vec{\nabla} + \frac{e}{\hbar c}\vec{A}(\vec{r}) + \frac{e}{\hbar c}\hat{\vec{a}}(\vec{r})]^2 \hat{\psi}_e(\vec{r}) + \int d^2\vec{r}\,' V_{Coul}(\vec{r}-\vec{r}\,')\hat{\rho}(\vec{r}\,')\hat{\psi}_e(\vec{r}) + \\
& + \frac{e}{c}\frac{d\hat{\omega}(\vec{r})}{dt}\hat{\psi}_e(\vec{r}) + \frac{e^2}{2\hbar c^2}\hat{L}(\vec{r})\hat{\psi}_e(\vec{r}); \\
& i\hbar \frac{d}{dt}\hat{\psi}_h(\vec{r}) = \frac{\hbar^2}{2m_h}[-i\vec{\nabla} - \frac{e}{\hbar c}\vec{A}(\vec{r}) - \frac{e}{\hbar c}\hat{\vec{a}}(\vec{r})]^2 \hat{\psi}_h(\vec{r}) - \int d^2\vec{r}\,' V_{Coul}(\vec{r}-\vec{r}\,')\hat{\rho}(\vec{r}\,')\hat{\psi}_h(\vec{r}) - \\
& \qquad - \frac{e}{c}\frac{d\hat{\omega}(\vec{r})}{dt}\hat{\psi}_h(\vec{r}) + \frac{e^2}{2\hbar c^2}\hat{L}(\vec{r})\hat{\psi}_h(\vec{r}); \\
& \frac{e^2}{2\hbar c^2}\hat{L}(\vec{r}) = \frac{\hbar^2\phi^2}{2m_e}\int d^2\vec{r}\,' \frac{\hat{\rho}_e(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^2} + \frac{\hbar^2\phi^2}{2m_h}\int d^2\vec{r}\,' \frac{\hat{\rho}_h(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^2}; \\
& \hat{\rho}(\vec{r}) = \hat{\rho}_e(\vec{r}) - \hat{\rho}_h(\vec{r}); \quad \hat{\omega}(\vec{r}) = -\frac{\phi e}{\alpha}\int d^2\vec{r}\,'\theta(\vec{r}-\vec{r}\,')\hat{\rho}(\vec{r}\,'); \quad \hat{\vec{a}}(\vec{r}) = \vec{\nabla}\hat{\omega}(\vec{r}).
\end{aligned}
\tag{37}
$$

It is evident that the composite electrons and holes have effective masses m_e and m_h and undergo the Landau quantization under the action of an external magnetic field and an effective magnetic field created by the vortices. It is a self-consistence in the system because the Landau quantization determines the quantum states of the composite particles; these quantum states, in turn, determine the value of the effective magnetic field. The electric charges of the composite particles are the same as those of the bare components and determine their Coulomb interactions and the interactions with the effective vector and scalar potentials created by the vortices. The kinetic energy of the vortices created with the participation of electrons and holes is also present in the Schrödinger equations.

4. Conclusions

The C–S gauge field created by quantum point vortices attached to electrons and holes situated on the lowest Landau levels under conditions of FQHEs gives rise to the vector and scalar potentials. The C–S vector potential generates an effective magnetic field depending on the difference between the density operators of electrons and holes in the way: $\hat{\rho}(\vec{r}) = \hat{\rho}_e(\vec{r}) - \hat{\rho}_h(\vec{r})$. In the mean field approximation, when this difference vanishes, the C–S effective magnetic and electric fields also vanish. In this case, the Landau quantization of the composite particles takes place under the action only of the external magnetic field. The effective masses of the composite particles are the same as those of the bare ones.

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