THE THREE KINDS OF BROWNIAN MOTION

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Abstract

In addition to the Brownian motion of a small particle in the air and the Brownian motion of a resonator, a third and new kind is proposed; it is referred to as the "coherent Brownian motion" and represents the permanent in-phase Brownian motions of an array of nanoresonators. From a theoretical point of view, the possibility of achieving that is addressed, and from a practical point of view, it is shown that the coherent Brownian motion is within current technology reach. An application in energy harvesting devices is considered.

1. Introduction

The description by Robert Brown in his article [1] of 1828, of what will later be called the Brownian motion, drew only little attention of physicists. Some of them supposed that its origin was molecular agitation [2]; however, one had to wait until 1905 and Albert Einstein's article [3] to see that the Brownian motion plays a more important role in Physics [4]. French physicist Jean Perrin conducted experimental tests of the Einstein's theory, which confirmed it and gave evidence of the existence of atoms [5].
Usually, large amplitude of Brownian motion is only observable on microscopic particles. Macroscopic objects do also have a Brownian motion, but so small that it is negligible, although still observable [6]. With the new kind of Brownian motion introduced in this study, large amplitudes of vibration, sustained by thermal agitation, appear in macroscopic bodies and give rise to unexpected properties.

2. The First Kind: the Brownian Motion of a Small Particle in a Fluid Medium

In Fig. 1, the position of a particle subjected to a Brownian motion is taken at a constant time interval, which gives this broken-line trajectory.

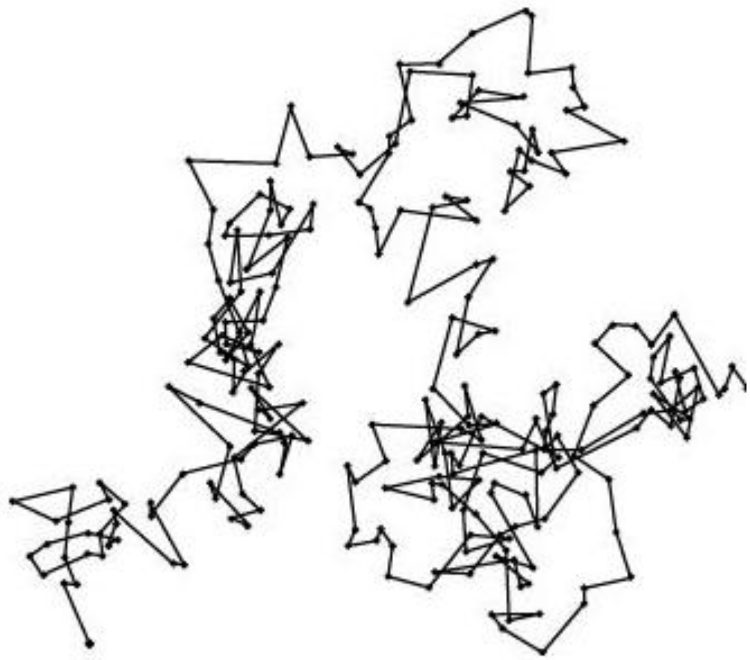

Fig. 1. Brownian motion of a particle (Jean Perrin [7]).

Mean-square displacement $\langle x^2 \rangle$ of the particle along the X-direction is [3, 5]

$$\langle x^2 \rangle = 2Dt \tag{1}$$

where x is the displacement of the particle, D is the diffusion constant, and t is the time.

The main feature of this first kind of Brownian motion is that it is a completely disordered motion that never stops.

3. The Second Kind: the Brownian Motion of a Resonator

The Brownian motion of a mechanical resonator [6, 8–19] related to a damped harmonic oscillator is described by the following Langevin equation:

$$m\frac{d^2x}{dt^2} + r\frac{dx}{dt} + kx = f_{th} \tag{2}$$

where x is the resonator displacement, t is the time, m is the resonator mass, r is the resonator mechanical resistance, k is the resonator spring constant, and f_{th} is the fluctuating force due to thermal agitation that drives the resonator. Spectral density $S_{f_{th}}(\omega)$ is constant over all frequencies corresponding to a broad band random process or white noise [19]:

$$S_{f_{th}}(\omega) = 4k_B Tr \tag{3}$$

where ω is the angular frequency, k_B is the Boltzmann's constant, and T is the absolute temperature in Kelvin.

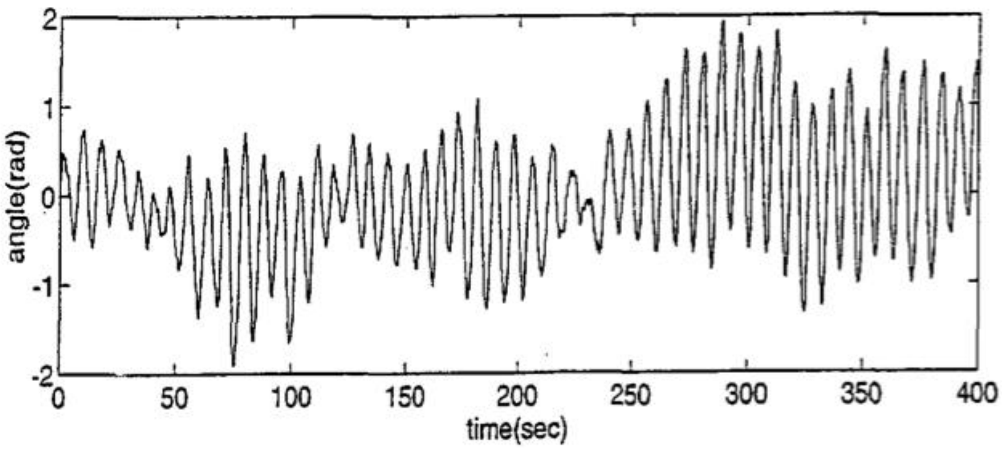

Fig. 2. Brownian motion of a resonator [16] (reproduced with the permission of G. I. González).

Figure 2 shows the Brownian motion of a resonator, which looks like a sine wave of varying amplitude and phase. It is a time record of resonator displacement x. The spectral density of x is peaking at the resonant angular frequency of the resonator ω_0 [12]. So x is a Gaussian narrow band random process [19], and the Brownian motion of a resonator is a vibration of steady frequency ω_0. Then it can be said that a resonator is a fascinating system, which has an outstanding characteristic: its Brownian motion is ordered. This has been well known for a long time [8], but it was not considered outstanding and enough attention has not been paid to it. I point out here this unique property, and I emphasize its prime importance in Physics, which has so far never been acknowledged.

The main feature of this second kind of Brownian motion is its fairly constant vibration frequency, and that it produces order from the disorder of thermal agitation.

4. The Third Kind: Coherent Brownian Motion

4.1. Introduction

Permanent in-phase Brownian motions are possible owing to a resonance, which occurs in mechanical interaction between a macroscopic structure—a plate—and a very large number of microscopic ones—nanopillars [20]. I propose to refer to this unknown kind of Brownian motion as the coherent Brownian motion. This new phenomenon allows energy to be extracted from Brownian motion without preventing such an extraction as stated by the Principle of Microscopic Reversibility [21, 22].

4.2. Description of the device

The device consists of two parts. The first part is a macroscopic object—a plate—which can be square or circular in shape, with a side or a diameter, respectively, of at least 1 mm.

The second part comprises a huge number of microstructures—nanopillars—which form an array on the plate (Fig. 3). An example of a pillar height is 1 μm and the pillar diameter is 100 nm. The aspect ratio (height/diameter) is 1 to 10.

The plate thickness is several times the pillar height.

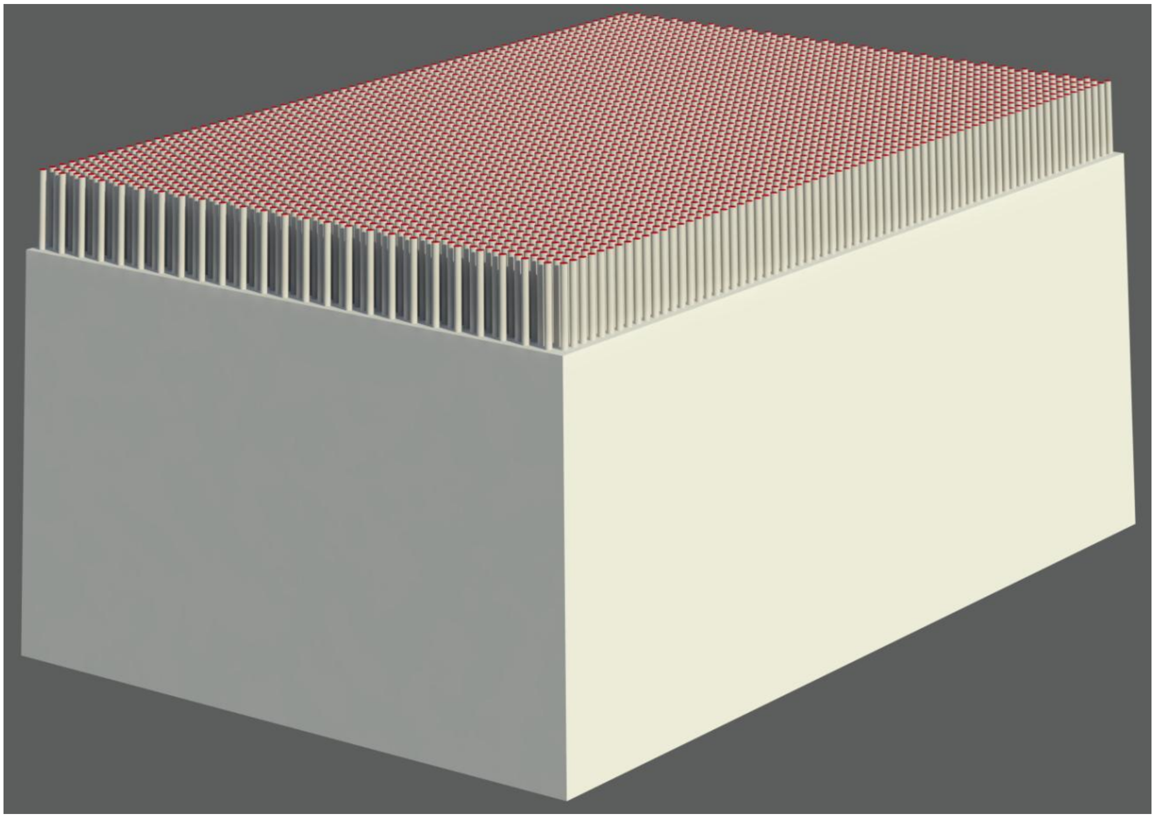

Fig. 3. Small part of the device (50 × 50 pillars), which actually comprises 10^8 to 10^{12} pillars or more.

4.3. The specific problem under study: the random phases of the Brownian motions of nanoresonators

4.3.1 Presentation of the problem: Nanopillars are tiny resonators subjected to Brownian motion; the vibration under consideration is a bulk wave one directed along the axis; it is referred to as the "length mode" (Fig. 4). In this device, the pillars of the array have the same resonant frequency, but random phases, since they vibrate independently of each other. Even if all pillars vibrated in phase at a given time, this process would not last because of the varying phase of the Brownian motion. Then, the question is to maintain one phase throughout the array.

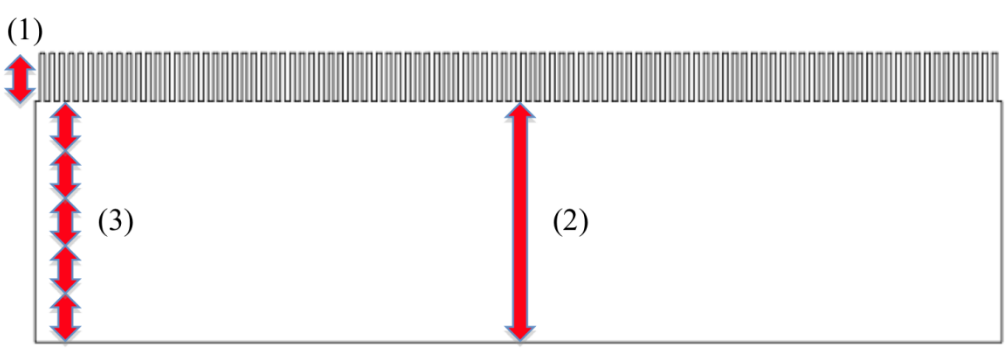

Fig. 4. Cross section of a small part of the device showing: (*1*) the length mode of the pillars, (*2*) the thickness compression mode of the plate, and (*3*) a harmonic of it.

4.3.2 The proposed solution: a mechanical resonance. In order to address this question, the plate is regarded as a macroscopic resonator. Its vibration is a thickness compression mode (Fig. 4), a harmonic of which has the same frequency as the pillar length mode. Then, as soon as the pillars vibrate in phase (ways to achieve this effect are proposed in the next paragraph), they will induce a large resonance of the plate. In turn, the plate, due to its big mass, will impose one single phase to the small pillars. To sum up, the pillars drive the plate, and the plate locks the phase of the pillars.

This makes a standing wave (Fig. 5) appear inside the device, with nodes of vibration and antinodes, including plate and pillars. The pink curve in Fig. 5 is the standing wave. The drawings below the curve show a small part of different device shapes.

The result of this mechanical resonance is a permanent vibration of the plate driven by the in-phase Brownian motions of the pillars.

4.4 Keeping up one phase in an array of nanopillars

In the following, the plate is made of a piezoelectric material with an electrode on each side. I propose two ways to keep up one phase in the vibrations of the pillars.

4.4.1 Application of an external voltage: An easy way to set the pillars in phase is to apply an alternating voltage to the electrodes at the resonant frequency during a short time. I make the supposition, according to paragraph 4.3.2, that the device will then maintain one phase in the vibrations of the pillars.

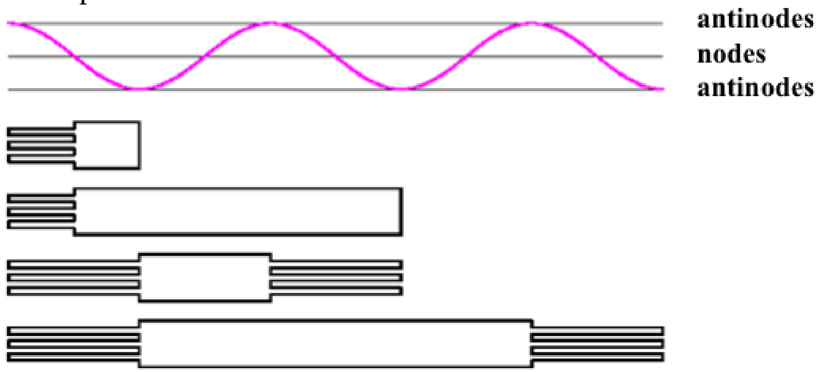

Fig. 5. Standing wave.

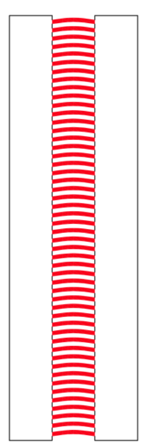

Fig. 6. Double-plate device.

4.4.2 The double-plate device: What is not a supposition but a certainty is the maintaining of the phase provided by a device comprising two plates and an array of nanopillars between those plates (Fig. 6). The latter are set in phase as in the previous case. The resonance of the two plates prevents any phase shift in the vibration of the pillars. The example chosen in Fig. 6 is a flexural vibration; however, it also works for the pillar length mode. The pillars are linked to the plates by their ends. However, they vibrate under thermal agitation. This is essentially because those pillars, on the one hand, have the size to be subjected to a large Brownian motion and, on the other hand, are free to vibrate for the specific mode under study, since their ends are nodes of vibration. Because of this freedom, they can experience Brownian motion.

4.5 Energy harvesting device

The constant vibration of the piezoelectric plate makes a permanent alternating voltage appear at the electrodes. Then, this device can supply an electrical power. At absolute temperature T, the mean energy of the pillar Brownian motion [11] is $k_B T$, the very low value of thermal energy. It is not possible to get a sufficiently large number n of pillars so that it can balance this value. For this reason, the energy of an array of n pillars, which is $nk_B T$, remains low. However, the array power is proportional [23] to both n and ω_0. Then it is product $n\omega_0$ that must balance the above value to get a reasonable amount of power:

$$n\omega_0 \approx (k_B T)^{-1}. \tag{5}$$

This is much easier than in the case with n alone and feasible with the present technology. In this case, the device output power is large enough to be useful.

4.6 Conditions for the resonance to occur

The main conditions for the appearance of resonance in the plate are as follows.

(i) An extremely large number of nanopillars.
(ii) An accurate tuning of the pillar resonant frequencies.
(iii) An accurate ratio of plate thickness to pillar height.
(iv) A not too low ratio of array mass to plate mass.
(v) Very small mechanical losses in the device or, in other words, a device with a very high quality factor.

4.7 A direct connection between the microscopic world and the macroscopic one

Innumerable Brownian motions, coming from the microscopic world, the world of atoms, molecules, micro and nanostructures, are gathered together to produce a mechanical resonance in the macroscopic world, our world. This provides a unique opportunity to observe, at the macroscopic scale, the behavior that usually is the privilege of very low dimensional objects. For instance, the permanent vibration of the plate, which comes from thermal agitation, through the nanopillar array, has an amplitude which is far beyond that of the plate under direct thermal agitation, that is to say, far beyond its own Brownian motion. Then a macroscopic object, the Brownian motion of which is imperceptible, exhibits a vibration due to thermal agitation, which has the same amplitude as the Brownian motion of a nanostructure, a very unusual and strange situation. This makes features that normally belong to microstructures, appear in macrostructures, and this unexpected and amazing property leads to new applications.

4.8 Fabrication

Current technology is able to produce arrays containing a very large number of nanopillars. The best way seems to be: to start with a piezoelectric film (the plate) deposited on a substrate (with electrodes), to deposit a thin film on the surface of this piezoelectric film, and to etch it by RIE or ICP [24–26] in order to get the array. After that, the piezoelectric film is made free-standing by removing the substrate.

5. Discussion and Conclusions

The main feature of this third kind of Brownian motion is a higher degree of order than that of a single resonator, mainly because of the vibration of the plate. The frequency of the pillar Brownian motions is very constant. There is no phase shift in them. There are still variations in the amplitude of the Brownian motion of a pillar, because the thermal energy still acts individually and separately on each pillar, but the plate resonance tends to diminish those changes. The time record of the vibration of any pillar of this array is closer to a sine wave than that in Fig. 2. These peculiar qualities immediately raise two questions:

1. Is coherent Brownian motion still a Brownian motion?
2. Does Physics allow coherent Brownian motion?

5.1 Is coherent Brownian motion still a Brownian motion?

Looking at the disorder in Fig. 1 and at the order in Fig. 2 would not incline us to think that, in both cases, this is Brownian motion, if we knew very little about it. Why then did physicists refer to this vibration of a resonator as Brownian motion? The answer is straightforward: thermal agitation is the cause of it and is also the cause of coherent Brownian motion. We have therefore to admit the possibility of occurrence of an unvarying-phase Brownian motion, the time record of which is close to a perfect sine wave, that is to say, a kind of Brownian motion exhibiting a higher degree of order than the two others.

On the other hand, the plate vibration, which is not a Brownian motion, although thermal energy keeps it going, has a time record which is a pure sine wave.

5.2 Does Physics allow coherent Brownian motion?

The nature of Brownian motion seems to have a varying phase in each pillar of an array. The nature of Brownian motion seems to have random phases in this array. Then I pose the question: how is it possible to assert that coherent Brownian motion will appear in this array, does Physics allow this?

The answer is yes, since there are examples of spontaneous in-phase thermal vibrations of molecules in some liquid crystals [27]. One explains the molecule shape leads to this collective or cooperative motion. If these molecules are taken to be small resonators, this fact strongly supports the existence of coherent Brownian motion.

5.3 Conclusions

We have therefore to acknowledge that the randomness of the usual Brownian motion phase is the result of events and circumstances, rather than a law. It is a contingent fact. For this reason, because randomness is not a fundamental property of Brownian motion, coherent Brownian motion will occur if the required experimental conditions are met.

The new phenomenon described in this study opens up a new path in the field not only of Brownian motion, but also of all random processes, suggesting that randomness is not a law, nor an essential feature of them, but the lack of any law. Since no law governs randomness, it is possible to introduce some order in random processes, if we know how to do that. I have given two examples of this: more order in the Brownian motion of a resonator, and more order even in thermal agitation by choosing molecules with a special shape without stopping being thermal agitation.

Finally, I would like to point out that coherent Brownian motion needs the device to be built and operate in order to become a reality.

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References

[1] R. Brown, Philosophical Magazine 4, 161, (1828).
[2] See for example: C. Wiener, Ann. Phys. (Leipzig) 118, 79 (1863); Gouÿ L. G., J. Phys. 7, 561, (1888); Rev. Gén. Sci. 1 (1895).
[3] A. Einstein, Annalen der Phys. 17, 549, (1905).
[4] S. G. Brush, The Kind of Motion We Call Heat, North-Holland, Amsterdam, Book 2, 1986.
[5] J. Perrin, Ann. Phys. Chim., 18, 5, (1909).
[6] P. F. Cohadon, A. Heidmann, and M. Pinard, Phys. Rev. Lett. 83, 3174 (1999).
[7] J. Perrin, Les atomes, Félix Alcan, Paris, 1913.
[8] G. E.Uhlenbeck and S. Goudsmit, Phys. Rev. 34, 145, (1929).
[9] M. Morimura, K. Nakagawa, and Y. Nezu, Jap. J. Appl. Phys. Suppl. 2, 135, (1974).
[10] V. B. Braginsky and A. B. Manukin, Measurement of Weak Forces in Physics Experiments, University of Chicago Press, Chicago, 1977.
[11] V. B. Braginsky, V. P. Mitrofanov, and V. I. Panov, Systems with small dissipation, University of Chicago Press, Chicago, 1985.
[12] P. R. Saulson, Phys. Rev. D 42, 2437, (1990).
[13] T. R. Albrecht, P. Grütter, D. Horne, and D. Rugar, J. Appl. Phys. 69, 668 (1991).
[14] T. B. Gabrielson, IEEE Trans. Electron Devices 40, 903, (1993).
[15] G. I. González and P. R. Saulson, Phys. Lett. A 201, 12, (1995).
[16] G. I. González, Brownian Motion of a Torsion Pendulum Damped by Internal Friction, Ph.D. thesis, Syracuse University, NY, 1995.
[17] E. Majorana and Y. Ogawa, Phys. Lett. A 233, 162, (1997).
[18] K. L. Ekinci and M. L. Roukes, Rev. Sci. Instrum. 76, 061101, (2005).
[19] D. E. Newland, An Introduction to Random Vibrations, Spectral & Wavelet Analysis, Third Edition, Dover Publications Inc., Mineola NY, 2005.
[20] J.-S. Rattinacannou, Keeping up One Phase in the Brownian Motions of Nanoresonators By Mechanical Meansm the 7th MSCMP Int. Conf., Chisinau, Republic of Moldova, 16-19 September 2014.
[21] G. E. Crooks, J. Stat. Mech.: Theory Exp. 7, P07008, (2011).

[22] R. Mauri, Non-Equilibrium Thermodynamics in Multiphase Flows (Chapter 2), 13-24, Springer, Dordrecht, The Netherlands, 2013.
[23] J.-S. Rattinacannou, EPJ Web of conferences, 79, 01006, (2014).
[24] Y.-F. Chang, Q.-R. Chou, J.-Y. Lin, and C.-H. Lee, Appl. Phys. A 86, 193, (2007).
[25] K. J. Morton, G. Nieberg, S. Bai, and S. Y. Chou, Nanotechnology 19, 345301, (2008).
[26] M. D. Henry, ICP Etching of Silicon for Micro and Nanoscale Devices, Ph.D. thesis, Caltech, CA, 2010.
[27] J. W. Goodby, Phase Structures of Calamitic Liquid Crystals, in Handbook of Liquid Crystals 2A, 12-13 Demus D. Ed., Wiley, New York, 1998.