

CALCULATION OF 2D-THERMOMAGNETIC CURRENT AND ITS FLUCTUATIONS USING THE METHOD OF EFFECTIVE HAMILTONIAN

R. G. Aghayeva

H. M. Abdullayev Institute of Physics, National Academy of Sciences of Azerbaijan,

H. Javid ave. 33, Baku, Az-1143 Azerbaijan

E-mail: a.rana@physics.ab.az

(Received 14 April 2010)

Abstract

The method of effective Hamiltonian (MEH) is proposed for the calculation of the nondissipative thermomagnetic current in samples of any dimension. This method includes the temperature in the Hamiltonian as opposed to the previous researches. The nondiagonal component of the thermomagnetic tensor calculated by the MEH is in full accord with results obtained previously. The calculations of the thermomagnetic current and its dispersion are performed in the coherent-state representation for the 2D-sample and nondegenerate statistics. The results obtained can be used, for example, to increase the sensitivity of temperature sensors.

1. Introduction

The advances in electronics, especially in nanoscale electronics, have attracted a wide interest in the study of noise in the electron devices, since noise restricts the sensitivity of devices operating with weak signals [1]. A noise is created by fluctuations. Fluctuations are generally small. However, they play the decisive role in the precise measurements and communication techniques [1]. In addition to the equilibrium fluctuations, there is a variety of the fluctuation mechanisms detected only in the process of a current flow. The simplest measure of the fluctuations, for example, of the current, is the current dispersion.

Currently known works contain, as a rule, a general analysis of noise (see, for example, [1]). The principal object of this paper is the calculation and estimation of the dispersion of nondissipative thermomagnetic current in the concrete 2D-sample. Namely, we considered a nondegenerate electron gas in a quantum well with the potential $U = m\omega_0^2 x^2 / 2$ and in a quantizing magnetic field $\vec{H} \parallel z$. Here m is the effective mass of electrons and ω_0 characterizes the parabolic potential. We supposed that the temperature gradient in the sample is directed along the x axis: $T = T(x)$.

To calculate the current dispersion d equal to

$$d = \langle \hat{j}^2 \rangle - \langle \hat{j} \rangle^2, \quad (1)$$

it is necessary to estimate the current density \hat{j} created by the electron.

Many authors tried to solve this problem via determining the thermoelectromotive force in a quantizing magnetic field [2]. However, in those papers, temperature was not included in the Hamiltonian, because temperature is related to the statistical force.

The method that includes temperature in the Hamiltonian (the method of effective Hamiltonian (MEH)) is proposed in [3] for 3D-sample. MEH makes it possible to calculate the thermomagnetic current and its dispersion immediately.

It is shown in present paper that MEH can be applied to calculate the thermomagnetic current in samples of any dimension, not only in 3D-samples.

The results obtained for d can be used, for example, to increase the sensitivity of temperature sensors. It should be noted that the dispersion of thermomagnetic current in the 3D-sample was calculated in [3].

2. The method of effective Hamiltonian

Let us suppose that the system under consideration is weakly nonuniform and $T=T(x)$. Then, the temperature deviation from its equilibrium magnitude is small and, in the simplest case of constant temperature gradient,

$$T(x) = T_0 \left[1 + \delta \left(xL_x^{-1} + \frac{1}{2} \right) \right] \quad (2)$$

Here L_x is the sample length along the x axis, $-(L_x/2) < x < (L_x/2)$, δ is the parameter of smallness, $\delta \ll 1$.

In the presence of an electric field \vec{E} , a temperature gradient ∇T , and a magnetic field \vec{H} the current density has the form

$$j_i = \sum_l \left[\sigma_{il} E_l^* - \beta_{il} \nabla_l T \right] \quad (3)$$

Here σ_{il} and β_{il} are the components of the conductivity tensors, $i, l = x, y, z$, $E_l^* = -\nabla(\varphi - \xi e^{-1})$, φ is the electrostatic potential, $-e$ is the electron charge, ξ is the chemical potential. On the assumption $T = T(x)$ and does not depend on y or z . Then, in the absence of the external electric field ($\varphi=0$) and $\xi=const$, we derive from (3) and (2)

$$j_y = -\beta_{yx} \partial T(x) / \partial x = -\beta_{yx} \delta T_0 / L_x . \quad (4)$$

We calculate this current, starting from the well known expression

$$j_y = -enTr\hat{\rho}\hat{v}_y, \quad (5)$$

where \hat{v} is the velocity operator, n is the density of conductivity electrons, and $\hat{\rho}$ is the nonequilibrium electron density matrix:

$$\hat{\rho} = Z^{-1} \exp \left[-(\hat{H} - \xi) / kT \right]. \quad (6)$$

Here Z is the partition function and k is the Boltzmann constant. In expression (6)

$$T^{-1} = T_0^{-1} \left[1 - \delta \left(\frac{x}{L_x} + \frac{1}{2} \right) \right] \quad (7)$$

is expanded in $\delta \ll 1$. Henceforth, we restrict ourselves to the first-order terms in δ . The Hamiltonian of the system can be represented as

$$\hat{H} = \hat{H}_0 - k\vec{r} \text{ grad} T . \quad (8)$$

The second term in equation (8) is connected with the inclusion of the temperature gradient effect

on the Hamiltonian of the equilibrium system \hat{H}_0 . By analogy with the electric field, we assume that the temperature is the potential of a certain external field with the intensity $-\text{grad } T$. The corresponding potential energy takes the form $-k\vec{r} \cdot \text{grad}T$, which in the case under consideration is reduced to $-kx\delta T_0/L_x$. Consequently, in constructing Hamiltonian (8), we proceed from the formal correspondence of the electrostatic potential φ with temperature T and the absolute value of the electron charge e with Boltzmann constant k .

Finally, instead of equation (6), we obtain

$$\hat{\rho} = Z^{-1} \exp\left[-(\hat{H}^* - \xi)/\gamma\right], \quad (9)$$

where

$$\gamma = (kT_0)^{-1}, \quad \hat{H}^* = \hat{H}_0 + \hat{V}, \quad (10)$$

$$\hat{V} = \delta \left(\frac{x}{L_x} + \frac{1}{2} \right) \xi - \frac{\delta}{\gamma} \frac{x}{L_x} - \frac{\delta}{2} \left(\hat{H}_0 \frac{x}{L_x} + \frac{x}{L_x} \hat{H}_0 + \hat{H}_0 \right). \quad (11)$$

The hermiticity of the operator \hat{V} is realized by the symmetrization of the product of the operators \hat{H}_0 and x .

Equation (9) shows that, in the presence of a small and uniform temperature gradient, the density matrix of the system is similar to that of the same system in the absence of the temperature gradient, but exposed to an external field whose contribution to the Hamiltonian of the system is given by the operator \hat{V} . It is clear from (11) that \hat{V} is a small perturbation, as it is proportional to the parameter of smallness δ . Hence, we expand density matrix (9) in a series using perturbation theory and restrict ourselves to a linear approximation of the parameter of smallness:

$$\hat{\rho} = \hat{\rho}_0 + \hat{\rho}_1. \quad (12)$$

Here $\hat{\rho}_0$ is the equilibrium density matrix:

$$\hat{\rho}_0 = Z^{-1} \exp\left[-(\hat{H}_0 - \xi)\gamma\right], \quad (13)$$

Z_0 is the corresponding statistical sum, and $\hat{\rho}_1$ is the nonequilibrium addition to the density matrix [4]:

$$\hat{\rho}_1 = \hat{\rho}_0 \int_0^\gamma d\gamma' \exp(\hat{H}_0\gamma') \hat{V} \exp(-\hat{H}_0\gamma'). \quad (14)$$

Starting from the well known expression for the velocity operator

$$\hat{v}_y = (i/\hbar) [\hat{H}, y], \quad (15)$$

we write equation (5) to first order in δ :

$$j_y = -(ien/\hbar) \text{Tr} \left(\hat{\rho}_0 [\hat{H}_0, y] + \hat{\rho}_0 [\hat{V}, y] + \hat{\rho}_1 [\hat{H}_0, y] \right). \quad (16)$$

Comparing equations (16) and (4), we can obtain the expression for the nondiagonal component of the thermomagnetic tensor β_{yx} .

Expression (16) is a general formula for the calculation of the transverse thermomagnetic current in the quantizing magnetic field.

The Hamiltonian of the equilibrium system \hat{H}_0 can be written as

$$\hat{H}_0 = \hat{h} + U, \quad (17)$$

where $\hat{h} = \frac{1}{2m} [\hat{p}_x^2 + (\hat{p}_y + m\omega_c x)^2 + \hat{p}_z^2]$, $\omega_c = eH/mc$ is the cyclotron frequency.

For the 1D-sample $U = m\omega_0^2(x^2 + z^2)/2$. This case was considered in [5].

For the 2D-sample $U = m\omega_0^2 x^2 / 2$.

For the 3D-sample $U = 0$. This case was considered in [3].

Calculated by the MEH, the nondiagonal component of the thermomagnetic tensor in [5, 3] is in full accord with results obtained previously [6, 7].

It should be noted that the MEH needs no account of the electron diamagnetism [7, 2]. It is easy to verify that the condition of $\xi = const$ does not affect the final result (16).

3. 2D-Thermomagnetic current

To calculate the 2D-thermomagnetic current \dot{j}_y (see (16)) we adopt the scheme from [3]. All the calculations in this paper are made on the basis of coherent states.

Following [3] we construct coherent states of \hat{H}_0 . For this we write \hat{H}_0 in the form

$$\hat{H}_0 = h + (m\omega_0^2 x^2)/2 = \hat{H}_\alpha + \hat{H}_\beta + \hat{H}_\gamma \quad (18)$$

where

$$\hat{H}_\alpha = \frac{1}{2m} [\hat{p}_x^2 + m^2 \omega^2 (x - \hat{x}_0)^2] = \hbar\omega \left(\hat{A}^+ \hat{A}^- + \frac{1}{2} \right), \quad (19)$$

$$\hat{H}_\beta = \left(\frac{\omega_0}{\omega} \right)^2 \frac{\hat{p}_y^2}{2m}, \quad \hat{H}_\gamma = \frac{\hat{p}_z^2}{2m}, \quad \omega^2 = \omega_c^2 + \omega_0^2, \quad \hat{x}_0 = - \left(\frac{\omega_c}{\omega} \right)^2 \frac{\hat{p}_y}{m\omega_c}, \quad (20)$$

$$\hat{A}^\mp = \sqrt{\frac{m\omega}{2\hbar}} \left[(x - \hat{x}_0) \pm \frac{i\hat{p}_x}{m\omega} \right] \exp(\pm i\omega t), \quad (21)$$

α, k_y, k_z are the quantum numbers. Then the solution of the wave equation

$\left(i\hbar \frac{\partial}{\partial t} - \hat{H}_0 \right) \psi = 0$ can be presented as

$$\psi = |\alpha\rangle |k_y\rangle |k_z\rangle. \quad (22)$$

Here

$$|k_y \rangle = \exp(ik_y y), \quad |k_z \rangle = \exp(ik_z z), \quad p_y = \hbar k_y. \quad (23)$$

$|\alpha\rangle$ satisfies all the necessary requirements of the coherent states [3] and equals

$$|\alpha\rangle = \sqrt{\frac{m\omega}{\pi\hbar}} \exp\left[-\left(\sqrt{\frac{m\omega}{2\hbar}}(x-x_0) - \alpha \exp(-i\omega t)\right)^2 + \frac{\alpha^2 \exp(-2i\omega t)}{2} - \frac{|\alpha|^2}{2}\right]. \quad (24)$$

To calculate the current, we adopt the scheme from [3]: express x and \hat{p}_x of the operators \hat{A}_α^\pm , arrange the operators, replace the operators by their eigenvalues; integrate over α, k_y, k_z using the standard integrals. We omit the terms proportional to the product $(\hat{A}_\alpha^+)^p (\hat{A}_\alpha^-)^s$ at $p \neq s$, to k_y^l and k_z^l at $l=1,3,5,\dots$, as they give zero when integrated over α, k_y, k_z , respectively.

The first term in (16) equals to zero when integrated over k_y . Making the cyclic permutation, we transform the second term in (16) to the form

$$\text{Tr} \hat{\rho}_0 [\hat{V}, y] = -\frac{\delta}{L_x} \text{Tr} \hat{\rho}_0 x [\hat{H}_0, y]. \quad (25)$$

Making use of equations (21) x can be expressed in terms \hat{A}_α^\pm :

$$x = \hat{x}_A + \hat{x}_0, \quad \hat{x}_A = \sqrt{\frac{\hbar}{2m\omega}} (e^{i\omega t} \hat{A}_\alpha^+ + e^{-i\omega t} \hat{A}_\alpha^-). \quad (26)$$

The integration over γ' in third term is reduced to

$$\text{Tr} \hat{\rho}_1 [\hat{H}_0, y] = -\text{Tr} \hat{\rho}_0 \gamma \delta(\zeta - \gamma^{-1} - \hat{H}_0) \hat{x}_0 L_x^{-1} [\hat{H}_0, y]. \quad (27)$$

It can be easily shown that, in the coherent-state representation,

$$[\hat{H}_0, y] = -\frac{i\hbar}{m} \left(\frac{\omega_0}{\omega}\right)^2 \hat{p}_y. \quad (28)$$

Finally, from equation (16) we derive

$$j_y = -\left(\frac{\omega_c}{\omega}\right)^2 \frac{nck}{H} \left(\frac{\xi}{kT_0} - \frac{\hbar\omega}{2kT_0} \coth \frac{\hbar\omega}{2kT_0} - 2\right) \frac{\delta T_0}{L_x}. \quad (29)$$

This expression is in complete agreement with the results of [8] obtained without the introduction of temperature in the Hamiltonian.

4. 2D-Thermomagnetic current dispersion

Let us calculate the dispersion of the 2D-thermomagnetic current in the linear approximation on the basis of equation (1).

Considering that the current $\langle \hat{j}_y \rangle$ is proportional to δ (see (29)), we omit $\langle \hat{j}_y \rangle^2$ in the

linear approximation. Then, based on expression (1), we write the dispersion of our system in the form

$$d = n^2 e^2 \text{Tr}(\hat{\rho} \hat{v}_y^2) . \quad (30)$$

Using the density matrix $\hat{\rho}$ from (12)-(14) and velocity operator (15) and (28), we find

$$d = \left(\frac{ne}{m}\right)^2 \left(\frac{\omega_0}{\omega}\right)^4 \text{Tr} \left[\hat{\rho}_0 (1 - \delta - \text{Tr} \hat{\rho}_1) \hat{p}_y^2 - 2 \frac{\delta}{L_x} \hat{\rho}_0 \hat{p}_y^2 x + \hat{\rho}_1 \hat{p}_y^2 \right]. \quad (31)$$

The member $\text{Tr} \hat{\rho}_1$ appears in the first term of the statistical sum expansion in the perturbation. The equilibrium statistical sum is widely used in equation (31).

The second term in the expression for the dispersion d tends to zero when integrated over α . It can easily be shown that

$$\text{Tr} \hat{\rho}_0 \hat{p}_y^2 = \frac{m}{\gamma} \left(\frac{\omega}{\omega_0} \right)^2 . \quad (32)$$

In this case,

$$d = d_0 + d_1 \quad (33)$$

where

$$d_0 = \frac{n^2 e^2}{m \gamma} \left(\frac{\omega_0}{\omega} \right)^2 \quad (34)$$

is the equilibrium part of the dispersion, while

$$d_1 = \left(\frac{ne}{m}\right)^2 \left(\frac{\omega_0}{\omega}\right)^4 \text{Tr} \hat{\rho}_1 \hat{p}_y^2 - d_0 \text{Tr} \hat{\rho}_1 - \delta d_0 \quad (35)$$

is the additional nonequilibrium part.

It is evident from expression (30) that we restrict ourselves to the dispersion due to the electron velocity fluctuation. The expression for d_0 can be obtained from d at $\delta = 0$, which corresponds to the equilibrium state of the system characterized by the temperature T_0 . Consequently, d_0 is related to the fluctuation due to the chaotic thermal motion of the particles forming the system (thermal noise).

If the conductor has a constant temperature gradient, this gives rise to an additional contribution d_1 to the thermomagnetic current dispersion d connected with the random character of the charge carrier diffusion.

Let us consider d_1 . Subsequent to arranging the operators, using the eigenvalues and integrating over α , k_y , k_z (see [5]) we obtain

$$\begin{aligned} \text{Tr} \hat{\rho}_1 &= \frac{\delta}{2} \left(1 - \xi \gamma + \frac{\hbar \omega \gamma}{2} \coth \frac{\hbar \omega \gamma}{2} \right), \\ \text{Tr} \hat{\rho}_1 \hat{p}_y^2 &= \frac{\delta m}{2 \gamma} \left(\frac{\omega}{\omega_0} \right)^2 \left(2 - \xi \gamma + \frac{\hbar \omega \gamma}{2} \coth \frac{\hbar \omega \gamma}{2} \right) \end{aligned} \quad (36)$$

Substituting (36) into (35) and using (34), we finally obtain the relation of the nonequilibrium part of the dispersion to the equilibrium part in the form

$$\frac{d_1}{d_0} = -\frac{\delta}{2} \quad (37)$$

Consider the longitudinal isothermal Nernst-Ettingshausen effect, i.e., the appearance of an electric field E along the temperature gradient in a solid conductor placed in a magnetic field which is perpendicular to the temperature gradient. It is known that, if the temperature gradient is directed along the x axis and the strong magnetic field along the z axis, then

$$E_x = -j_y \sigma_{yx}^{-1}, \quad (38)$$

$\sigma_{yx} = nceH^{-1}$ is the expression for the nondiagonal component of the conductivity tensor. The quantity E is conveniently expressed in terms of the voltage u :

$$u = -j_y \sigma_{yx}^{-1} L_x. \quad (39)$$

Taking into account the thermomagnetic current dispersion, we obtain the mean-square noise voltage in the form

$$\sqrt{\langle u^2 \rangle} = -d^{1/2} \sigma_{yx}^{-1} L_x. \quad (40)$$

This noise limits the sensitivity of the temperature sensor based on the longitudinal isothermal Nernst-Ettingshausen effect. The input signal is really indistinguishable from the background of

random vibrations if it is less than $\sqrt{\langle u^2 \rangle}$.

It is easy to verify that the ratio of noise voltage (40) to the noise voltage (40) at $\delta = 0$ is $[1 - (\delta/4)]$, and the term $(-\delta/4)$ is associated with correction d_1 . It is clear that the correction d_1 decreases the noise voltage. Therefore, we should consider the contribution from d_1 to the temperature sensors while performing measurements in a small temperature range but with sufficiently high accuracy.

I wish to express my sincere thanks to Prof. F.M. Hashimzade for helpful discussion.

References

- [1] M.Di. Ventra, Electrical Transport in Nanoscale Systems, Cambridge University Press, N-Y., 2008.
- [2] B.M. Askerov, Electron Transport Phenomena in Semiconductors, World Scientific, 1994.
- [3] R.G. Agayeva, J. Phys. C: Solid State Phys. 18, 5841(1985).
- [4] R.P. Feynman, Statistical Mechanics. W.A.Benjamin, Inc., Massachusetts, 1972.
- [5] R.G. Agayeva, Fizika XV, nos. 4, 3 (2009).
- [6] M.D. Bloch, Fiz. Tverd. Tela 17, 896 (1975).
- [7] Yu.N. Obraztsov, Fiz. Tverd. Tela 7, 573 (1965).
- [8] F.M. Hashimzade and Kh.A. Hasanov, Izvestiya NANA, seriya FMTN, XXVII, nos. 5, 3 (2007).