

MHD FREE CONVECTION FLOW NEAR A MOVING VERTICAL PLATE IN THE PRESENCE OF THERMAL RADIATION: AN ANALYTICAL SOLUTION

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Abstract

An exact solution of the problem of the unsteady free convection flow of a viscous electrically conducting incompressible fluid in the presence of thermal radiation and under the influence of uniform magnetic field applied normal to the plate near an infinite vertical plate, which moves at a time dependent velocity, is presented here by the Laplace transform technique. The fluid is also assumed to be gray; emitting absorbing but non scattering medium and the optically thick radiation limit is considered. The solutions of the present problem were obtained in closed forms with arbitrary magnetic field parameter (M), radiation parameter (F), Prandtl number (P_r), thermal Grashof number (G_r), etc. Some important applications of physical interest for different type motion of the plate are discussed. Applying numerical values into the expressions of analytical solution, we also discussed the vertical air flows — the usual phenomenon at plumes into the atmosphere.

1. Introduction

Free convection flow involving heat transfer occurs frequently in an environment where differences between land and air temperatures can give rise to complicated flow patterns. The subject of magnetohydrodynamics has attracted the attention of a large number of scientists due to its diverse application. In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere, etc. In engineering, it finds its application in MHD pumps, MHD bearing, etc. The study of effects of magnetic field on free convection flow is important in liquid metals, electrolytes, and ionized gasses. At the high temperature attained in some engineering devices, gas, for example, can be ionized and so becomes an electrical conductor. The ionized gas or plasma can be made to interact with the magnetic field and alter heat transfer and friction characteristic. Recently, it is of great interest to study the effect of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing thermal radiation. Heat transfer by thermal radiation is becoming of greater importance when we are concerned with space applications, higher operating temperatures and also power engineering.

Extensive research work has been obtained on free convection flow near vertical plate or surface with different boundary conditions [1, 2]. Free convection effects on flow past a vertical surface were studied by Vedhanayagam et al. [3], Kolar et al. [4], Soundalgekar [5],

and Revankar [6] with different boundary conditions. Many scientists, like Sahoo et al. [7] and Soundalgekar [8], worked in hydro-magnetic natural convection flow past a vertical surface. Several investigations have been carried out on problem of heat transfer by radiation as an important application of space and temperature related problems. Grief et al. [9] obtained an exact solution for the problem of laminar convective flow in a vertical heated channel in the optically thin limit. In the optically thin limit, the fluid does not absorb its own emitted radiation which means that there is no self-absorption but the fluid absorbs the radiation emitted by the boundaries. The forced convective flow in a horizontal channel permeated by uniform vertical magnetic field, taking radiation into account, was considered by Viskanta [10]. Gupta et al. [11] investigated the effect of radiation on the combined free and forced convection of an electrically conducting fluid flowing inside an open ended vertical channel in the presence of uniform magnetic field for the case of optically thin limit. Soundalgakar et al. [12] generalized the problem by considering the effect of radiation on the natural convection flow of a gas past a semi-infinite plate using the Cogly Vincentine Gilles equilibrium model (Cogly et al. [13]). Later, Takhar [14] and Hossian et al. [15] analyzed the effect of radiation using the Rosseland diffusion approximation for mixed convection of an optically dense viscous incompressible fluid in the presence of magnetic field. Hossain et al. [16] discussed the effect of radiation on free convection from a porous vertical plate. Muthcumaraswamy et al. [17] investigated the heat and mass transfer effects on moving vertical plate in the presence of thermal radiation. Recently, Majumder et al. [18] gave an exact solution for MHD flow past an impulsively started infinite vertical plate in the presence of thermal radiation.

The purpose of the present paper is to solve analytically the problem of the unsteady free convection flow of an optically thin viscous, electrically conducting incompressible fluid near an infinite vertical plate which moves at time dependent velocity in the presence of transverse uniform magnetic field and thermal radiation. A general exact solution for the partial differential equation governing these flows is obtained with the aid of the Laplace transform technique. Furthermore, this general solution is applied for the most important cases of flow.

2. Formulation of the problem

We consider an unsteady free convection flow of a viscous incompressible electrically conducting and radiating fluid along an infinite non-conducting vertical flat plate (or surface) in the presence of a uniform transverse magnetic field B_0 applied in the direction of the flow. On this plate, an arbitrary point has been chosen as the origin of a co-ordinate system with x' - axis along the plate in the upward direction and the y' - axis normal to the plate (Fig. 1).

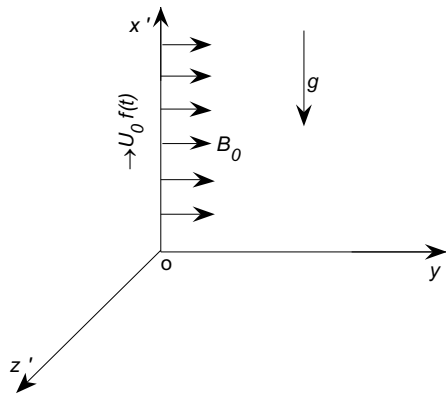


Fig. 1 – The physical model of the problem and coordinate system.

Initially for time $t' \leq 0$, the plate and the fluid are at the same constant temperature T'_∞ in a stationary condition. Subsequently ($t' > 0$), the plate is assumed to be accelerating with a velocity $U_0 f(t')$ in its own plane along the x' -axis; instantaneously, the temperature of the plate is raised to T'_w , which is hereafter regarded as constant. For free convection flows, here we also assume that all the physical properties of the fluid are in the direction of the x' -axis, so the physical variables are functions of the space co-ordinate y' and time t' only.

Under the above assumptions, the fully developed flow of a radiating gas is governed by the following set of equations

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) - \frac{\sigma B_0^2}{\rho} u', \quad (1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'}. \quad (2)$$

The initial and boundary conditions are

$$\left. \begin{aligned} u' = 0, T' = T'_\infty \text{ for all } y' \geq 0, t' \leq 0 \\ u' = U_0 f(t'), T' = T'_w \text{ at } y = 0 \\ u' = 0, T' = T'_\infty \text{ as } y \rightarrow \infty \end{aligned} \right\} t' > 0, \quad (3)$$

where u' is the velocity in the x' -direction, ρ is the density, g is the acceleration due to gravity, β is the coefficient of thermal expansion, T' is the temperature of the fluid near the plate, C_p is the specific heat at constant pressure, k is the thermal conductivity, q_r is the radiative flux, ν is the kinematic viscosity, and σ is the electrical conductivity.

In the optically thick limit, the fluid does not absorb its own emitted radiation; that is, there is no self-absorption but it absorbs the radiation emitted by the boundaries. It has been shown by Cogly et al. [13] that in the optically thick limit for a non gray gas near equilibrium

$$\frac{\partial q_r}{\partial y'} = 4(T' - T'_\infty) \int_0^\infty K_{\lambda\varpi} \left(\frac{de_{b\lambda}}{dT'} \right) d\lambda = 4I_1(T' - T'_\infty), \quad (4)$$

where $K_{\lambda\varpi}$ is the absorption coefficient, $e_{b\lambda}$ is the Planck function, and the subscript ϖ refers to values at the wall.

To reduce the above equations into non-dimensional form, let us introduce the following dimensionless variables and parameters

$$\left. \begin{aligned} u = \frac{u'}{U_0}, \quad t = \frac{t' U_0^2}{\nu}, \quad y = \frac{y' U_0}{\nu}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty} \\ M = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, \quad P_r = \frac{\mu C_p}{k}, \quad G_r = \frac{\nu g \beta (T'_w - T'_\infty)}{U_0^3}, \quad F = \frac{4I_1 \nu^2}{k U_0^2} \end{aligned} \right\} \quad (5)$$

where G_r is the thermal Grashof number, M is the magnetic field parameter, P_r is the Prandtl number, F is the radiation parameter.

With the help of (5), governing equations (1) and (2) are reduced to

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r \theta - Mu, \quad (6)$$

$$P_r \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - F \theta. \quad (7)$$

The corresponding initial and boundary conditions in non-dimensional form are

$$\left. \begin{aligned} u &= 0, \theta = 0 \text{ for all } y \geq 0, t \leq 0 \\ u &= f(t), \theta = 1 \text{ at } y = 0 \\ u &\rightarrow 0, \theta \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\} t > 0 \quad (8)$$

3. Solution of the problem

In order to obtain the analytical solution of the system of differential equations (6) and (7) subject to the initial and boundary conditions (8), we shall use the Laplace transformation technique. Thus, the general solutions of the present problem for the temperature $\theta(y, t)$ and the velocity $u(y, t)$ for $t > 0$ are given by

$$\theta(y, t) = \frac{1}{2} \left[e^{-y\sqrt{F}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{P_r}{t}} - \sqrt{\frac{Ft}{P_r}} \right) + e^{y\sqrt{F}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{P_r}{t}} + \sqrt{\frac{Ft}{P_r}} \right) \right], \quad (9)$$

$$u(y, t) = \Phi(y, t) + A(y, t), \quad (10)$$

where

$$\Phi(y, t) = L^{-1} [\bar{f}(s) e^{-ay}], \quad \bar{f}(s) = L[f(t)], \quad (11)$$

$$\begin{aligned} A(y, t) = & \frac{G_r}{2a} e^{ct} \left[e^{-y\sqrt{b}} \left\{ \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{bt} \right) - \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{P_r}{t}} - \sqrt{\frac{bt}{P_r}} \right) \right\} + \right. \\ & \left. e^{y\sqrt{b}} \left\{ \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{bt} \right) - \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{P_r}{t}} + \sqrt{\frac{bt}{P_r}} \right) \right\} \right] + \\ & \frac{G_r}{2a} \left\{ e^{-y\sqrt{F}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{P_r}{t}} - \sqrt{\frac{Ft}{P_r}} \right) + e^{y\sqrt{F}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{P_r}{t}} + \sqrt{\frac{Ft}{P_r}} \right) \right\} - \\ & \frac{G_r}{2a} \left\{ e^{-y\sqrt{M}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{Mt} \right) + e^{y\sqrt{M}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{Mt} \right) \right\}. \end{aligned} \quad (12)$$

and $a = M - F$, $b = \frac{MP_r - F}{P_r - 1}$, $c = \frac{M - F}{P_r - 1}$.

Since non-dimensional temperature $\theta(y, t)$ is clearly described in (9), so we shall confine ourselves to non-dimensional velocity $u(y, t)$ for various types of $f(t)$.

4. Application of the general solution

In this section, we now consider some important cases of flow as given below:

Case(i): Motion of the plate with uniform velocity

Let $f(t) = H(t)$, Heaviside unit function.

$$\therefore \bar{f}(s) = L[f(t)] = \frac{1}{s}$$

In this case, we observe that the result of (9) is unaffected and expression (10) for $u(y, t)$ becomes

$$u(y, t) = \frac{1}{2} \left\{ e^{\sqrt{M}y} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{Mt} \right) + e^{-\sqrt{M}y} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{Mt} \right) \right\} + A(y, t), \quad (13)$$

where $A(y, t)$ is given from equation (12).

Case (ii): Motion of the plate with single acceleration

Let $f(t) = tH(t)$, then $\bar{f}(s) = 1/s^2$.

In this case, we also observe that the result of (9) remains in the same form but expression (10) for $u(y, t)$ takes the following analytical form

$$u(y, t) = \frac{1}{2} \left\{ \left(t + \frac{y}{2\sqrt{M}} \right) e^{y\sqrt{M}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{Mt} \right) + \left(t - \frac{y}{2\sqrt{M}} \right) e^{-y\sqrt{M}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{Mt} \right) \right\} + A(y, t), \quad (14)$$

where $A(y, t)$ is given from (12).

Case (iii): Motion of the plate with periodic acceleration

For this case, let $f(t) = H(t) \cos \omega t$

$$\therefore \bar{f}(s) = \frac{s}{s^2 + \omega^2}.$$

Then expression (9) remains again in the same form but, instead of (10), we get the following analytical expression

$$u(y, t) = \frac{1}{4} e^{i\omega t} \left\{ e^{\sqrt{M+i\omega}y} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{(M+i\omega)t} \right) + e^{-\sqrt{M+i\omega}y} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{(M+i\omega)t} \right) \right\} + \frac{1}{4} e^{-i\omega t} \left\{ e^{\sqrt{M-i\omega}y} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{(M-i\omega)t} \right) + e^{-\sqrt{M-i\omega}y} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{(M-i\omega)t} \right) \right\} + A(y, t), \quad (15)$$

where the term $A(y, t)$ is given from (12).

It should be noted that our results of case (i) are identical with those of Mazumdar and Deka [18].

5. Skin friction

Knowing the velocity field, we now study the effect of t , P_r , M , F , etc. on the skin friction. In the non-dimensional form, it is given by

$$\begin{aligned} \tau &= - \left(\frac{\partial u}{\partial y} \right)_{y=0} \\ &= - \left(\frac{\partial \Phi}{\partial y} \right)_{y=0} + \frac{G_r}{a} e^{ct} \left[\sqrt{b} \left(\operatorname{erf} \sqrt{bt} - \operatorname{erf} \sqrt{\frac{bt}{P_r}} \right) + \frac{1}{\sqrt{\pi t}} e^{-bt} - \sqrt{\frac{P_r}{\pi t}} e^{-\frac{bt}{P_r}} \right] - \\ &\quad \frac{G_r}{a} \left(\sqrt{M} \operatorname{erf} \sqrt{Mt} + \frac{1}{\sqrt{\pi t}} e^{-Mt} \right) + \frac{G_r}{a} \left(\sqrt{F} \operatorname{erf} \sqrt{\frac{Ft}{P_r}} + \sqrt{\frac{P_r}{\pi t}} e^{-\frac{Ft}{P_r}} \right). \end{aligned} \quad (16)$$

When the plate is moving with uniform velocity, then

$$\left(\frac{\partial \Phi}{\partial y} \right)_{y=0} = - \left\{ \sqrt{M} \operatorname{erf} \sqrt{Mt} + \frac{1}{\sqrt{\pi t}} e^{-Mt} \right\}. \quad (17)$$

Again, when the plate is moving with single acceleration, then

$$\left(\frac{\partial \Phi}{\partial y}\right)_{y=0} = -\left\{\sqrt{M}\left(t + \frac{1}{2M}\right) \operatorname{erf} \sqrt{Mt} + \sqrt{\frac{t}{\pi}} e^{-Mt}\right\}. \quad (18)$$

Lastly, when the plate is moving with periodic acceleration, then

$$\begin{aligned} \left(\frac{\partial \Phi}{\partial y}\right)_{y=0} = & -\frac{1}{2\sqrt{t}} \left\{ e^{i\omega t} \sqrt{(M+i\omega)t} \operatorname{erf} \sqrt{(M+i\omega)t} + \right. \\ & \left. e^{-i\omega t} \sqrt{(M-i\omega)t} \operatorname{erf} \sqrt{(M-i\omega)t} \right\} - \sqrt{\frac{t}{\pi}} e^{-Mt}. \end{aligned} \quad (19)$$

6. Nusselt number

An important phenomenon in this study is to understand the effect of t, F on the Nusselt number. In non-dimensional form, the rate of heat transfer is given by

$$\begin{aligned} N_u = & -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \\ = & \sqrt{M} \operatorname{erf} \sqrt{\frac{Ft}{P_r}} + \sqrt{\frac{P_r}{\pi t}} e^{-\frac{Ft}{P_r}}. \end{aligned} \quad (20)$$

7. Numerical results and discussion

In order to get physical insight into the present problem, we give an example of evaluation of the numerical values of the velocity $u(y, t)$ and the temperature $\theta(y, t)$ for the case of air ($P_r = 0.71$). Numerical calculations in the present example have been carried out for different values of G_r, M, F, t and for fixed values of $P_r (= 0.71)$.

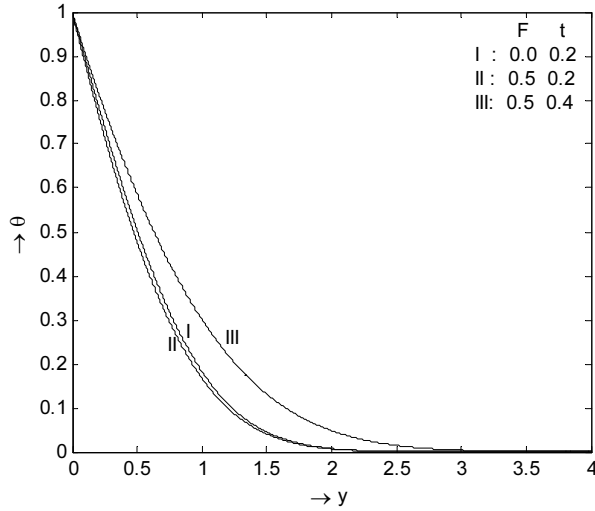


Fig. 2. Temperature profile (taking $P_r = 0.71$).

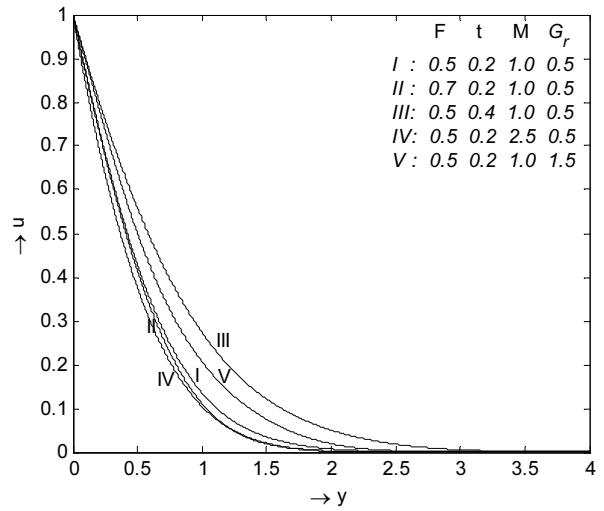


Fig. 3. Velocity profile when the plate moves at a uniform velocity (taking $P_r = 0.71$).

The temperature profiles are derived from (9) and are shown in Fig. 2 for air. We observe that the temperature decreases with increasing F and increases with increasing t .

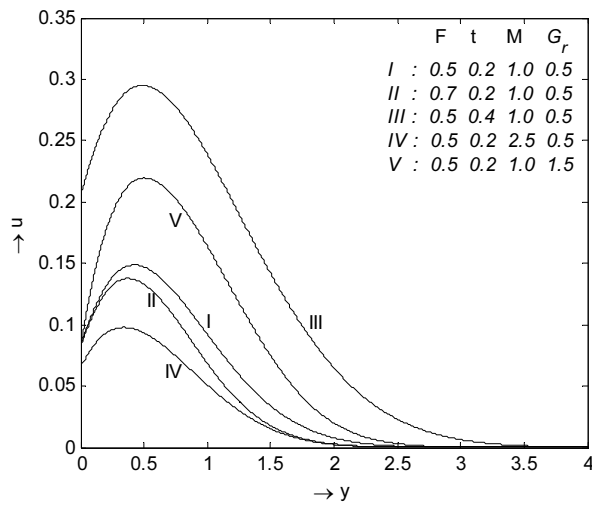


Fig. 4. Velocity profile when the plate moves with single acceleration (taking $P_r = 0.71$).

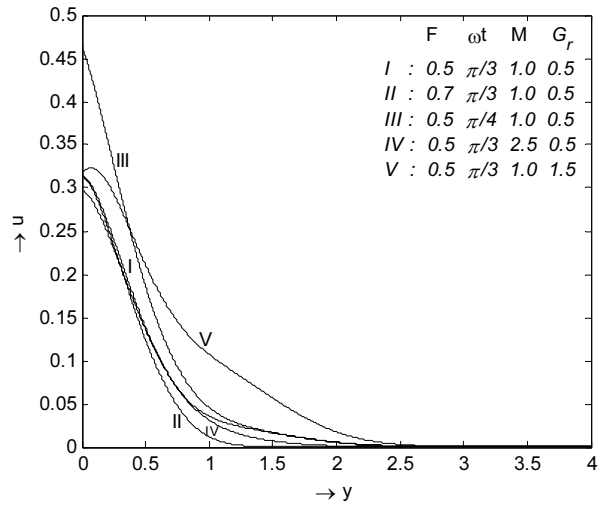


Fig. 5. Velocity profile when the plate moves with periodic acceleration (taking $P_r = 0.71$).

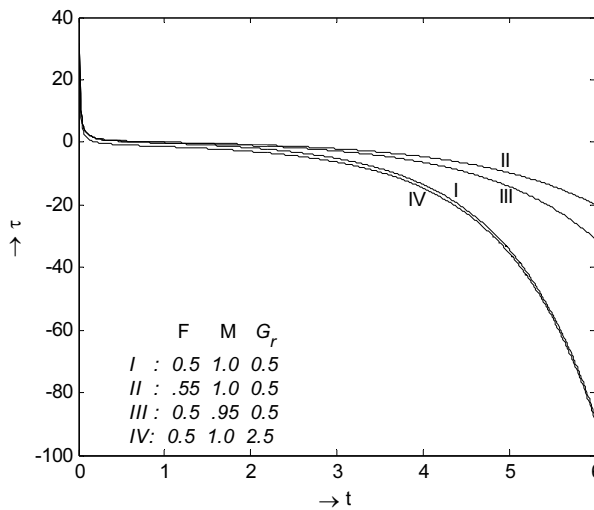


Fig. 6. Skin friction (for uniform velocity).

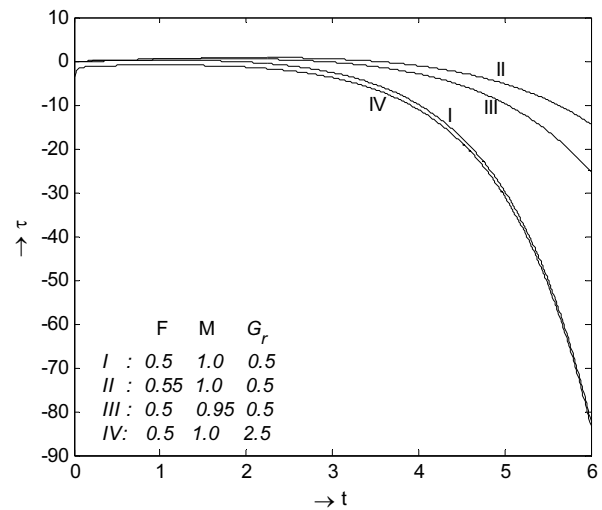


Fig. 7. Skin friction (for single acceleration).

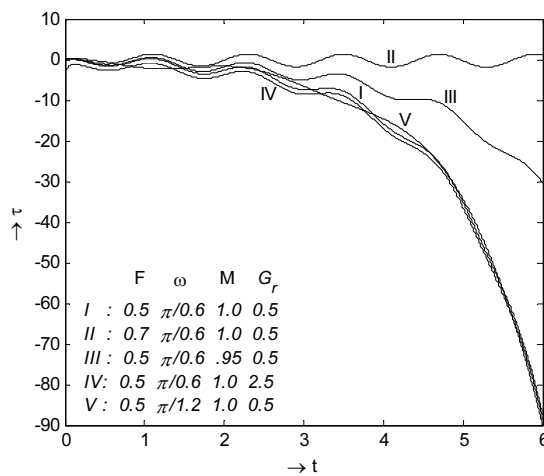


Fig. 8. Skin friction (for periodic acceleration).

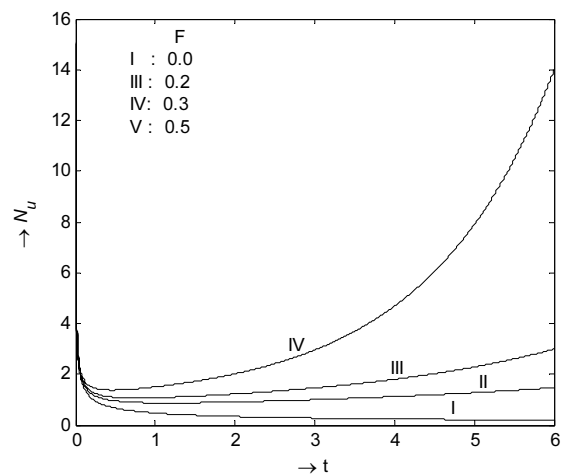


Fig. 9. Nusselt number (taking $P_r = 0.71$).

Applying numerical values into the expressions of exact solutions (cf. equations (10), (12), (13), (14), and (15)) for the velocity, we get the velocity profiles of air flows near vertical plate. Figures 3 and 4 correspond to the plates moving at a uniform velocity and with single acceleration, respectively. Figure 5 also corresponds to the plates, which move with periodic acceleration. It is seen from these figures that increasing values of M and F lead to a fall in velocity but an increase of t leads to an increase in the velocity of air.

Figures 6, 7, and 8 depict skin-friction against time t for different values of parameters. It is observed that the skin friction decreases with increasing M , F , and t . Thus, wall shear stress decreases with increasing radiation parameter, magnetic field parameter, and time.

Nusselt number is presented in Fig. 9 against time. It decreases with time but increases with increasing radiation parameter.

8. Conclusions

In this paper, the effect of thermal radiation on MHD free convection flow near a moving vertical plate is presented. A general analytical solution for the problem is determined without any restriction. Some important applications from the point of view of physical interest are discussed. Also, we study a physical example for evaluation of the numerical values of the velocity and the temperature for the case of air ($P_r = 0.71$) and observe that the temperature decreases with increasing F and increases with increasing t . Also, the velocity decreases with increasing F and M but increases with increasing time t . To our knowledge, this work gives, in a close form, the actual analytical solution of the MHD free convection flow of an optically thin fluid which has wide application in power engineering and also in the study of vertical air flows into the atmosphere.

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