BOSE-EINSTEIN CONDENSATION OF TWO-DIMENSIONAL MAGNETOEXCITONS ON THE SUPERPOSITION STATE AND EXCITED LANDAU LEVELS

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Abstract

The Bose-Einstein Condensation (BEC) of the two-dimensional magnetoexcitons on the superposition state, as well as on the excited Landau levels, is studied. The superposition of two excitonic states formed by electron and hole on the lowest Landau levels (0, 0) and on the first excited Landau levels (1, 1) was considered. The generalized Bogoliubov *u-v* transformation was deduced. The criterion for the exciton concentration was established. These questions will be studied below.

1. Introduction

In Refs [1, 2] the influence of the Coulomb scattering processes of electrons and holes between different Landau levels on the energy spectrum and on the collective properties of the two-dimensional magnetoexcitons was investigated. It was shown that the virtual quantum transitions of the Coulomb interacting particles from the lowest Landau levels to excited Landau levels with arbitrary numbers *n* and *m* and their transitions back to the lowest Landau levels in the second order of the perturbation theory result in the indirect attraction between the particles supplementary to their Coulomb interaction. These investigations were fulfilled in pure electron-hole description as well as in the excitonic approach, which permitted to determine the wave function of the lowest magnetoexciton band. The more exact exciton wave function, as well as the exciton creation operator, was represented in the superposition form composed by the wave function and creation operators of the bare exciton states. The existence of the ground state in the form of superposition state will influence the character of the magnetoexciton BEC, changing the distribution of the particles on the lowest and excited Landau levels.

2. Bose-Einstein Condensation of two-dimensional magnetoexcitons on the superposition state

As was shown in references [2, 3] the ground state of the 2D magnetoexciton in a simplified case can be described by the exciton creation operator

$$\hat{X}^{\dagger}(\vec{k}) = \xi \hat{X}^{(0,0)\dagger}(\vec{k}) + \eta \hat{X}^{(1,1)\dagger}(\vec{k}); \quad \xi^2 + \eta^2 = 1, \tag{1}$$

where the creation operators $\hat{X}^{(0,0)\dagger}(k)$ and $\hat{X}^{(1,1)\dagger}(k)$ describe the bare magnetoexcitons states, first of them being created by the electron and hole on the lowest Landau levels (LLLs) with $n_e = n_h = 0$, whereas the second exciton state being created by the first excited Landau levels (FELL) $n_e = n_h = 1$.

To introduce the Bose-Einstein condensation (BEC) state of excitons, as was shown in many previous references [4, 5], it is necessary to use the operator of the unitary transformation

$$\hat{D}\left(\sqrt{N_{ex}}\right) = e^{\hat{Y}}; \quad \hat{Y} = \sqrt{N_{ex}}\left(X^{\dagger}(k) - X(k)\right), \tag{2}$$

where N_{ex} is the total number of the excitons and $X^{\dagger}(k)$, $X(k)$ are the single exciton creation and annihilation operators. In our case of a superposition state (1), the operator $\hat{Y}$ can be transcribed as

$$\hat{Y} = \sqrt{N_{ex}}\left[\xi\left(\hat{X}^{(0,0)\dagger}(k) - \hat{X}^{(0,0)}(k)\right) + \eta\left(\hat{X}^{(1,1)\dagger}(k) - \hat{X}^{(1,1)}(k)\right)\right] = \hat{Y}_0 + \hat{Y}_1. \tag{3}$$

Introducing the filling factor v^2 in the form $N_{ex} = v^2 N$, where $N = S/2\pi l^2$ and taking into account the definitions of the operators $\hat{X}^{(0,0)\dagger}(k)$ and $\hat{X}^{(1,1)\dagger}(k)$ we can represent $\hat{Y}_0$ and $\hat{Y}_1$ operators as

$$\begin{aligned} \hat{Y}_0 &= v\xi\sum_t \left[e^{-ik_y t l^2} a^{\dagger}_{t+\frac{k_x}{2}} b^{\dagger}_{-t+\frac{k_x}{2}} - e^{ik_y t l^2} b_{-t+\frac{k_x}{2}} a_{t+\frac{k_x}{2}} \right] \\ \hat{Y}_1 &= v\eta\sum_t \left[e^{-ik_y t l^2} c^{\dagger}_{t+\frac{k_x}{2}} d^{\dagger}_{-t+\frac{k_x}{2}} - e^{ik_y t l^2} d_{-t+\frac{k_x}{2}} c_{t+\frac{k_x}{2}} \right] \end{aligned} \tag{4}$$

They have the properties

$$\hat{Y}_0^{\dagger} = -\hat{Y}_0; \quad \hat{Y}_1^{\dagger} = -\hat{Y}_1; \quad \hat{Y}^{\dagger} = -\hat{Y}; \quad \left[\hat{Y}_0, \hat{Y}_1\right] = 0, \tag{5}$$

which lead to the expressions

$$\begin{gathered} \hat{D} = \hat{D}_0\hat{D}_1, \quad \left[\hat{D}_0, \hat{D}_1\right] = 0; \quad \hat{D}^{\dagger} = \hat{D}^{-1} = e^{-\hat{Y}} \\ \hat{D}_0 = e^{\hat{Y}_0}, \hat{D}_1 = e^{\hat{Y}_1}. \end{gathered} \tag{6}$$

Now we will transform the initial electron and hole creation and annihilation operators $a_p^{\dagger}$, a_p, $b_p^{\dagger}$, b_p, $c_p^{\dagger}$, c_p and $d_p^{\dagger}$, d_p corresponding to the Landau quantization states $n_e, n_h = 0, 1$ as follows

$$\begin{aligned} \hat{D}a_p\hat{D}^{\dagger} &= \alpha_p = u_0 a_p - v_0 e^{-ik_y(p-k_x/2)l^2} b^{\dagger}_{k_x-p} \\ \hat{D}b_p\hat{D}^{\dagger} &= \beta_p = u_0 b_p + v_0 e^{ik_y(p-k_x/2)l^2} a^{\dagger}_{k_x-p} \\ \hat{D}c_p\hat{D}^{\dagger} &= \gamma_p = u_1 c_p - v_1 e^{-ik_y(p-k_x/2)l^2} d^{\dagger}_{k_x-p} \\ \hat{D}d_p\hat{D}^{\dagger} &= \delta_p = u_1 d_p + v_1 e^{ik_y(p-k_x/2)l^2} c^{\dagger}_{k_x-p}, \end{aligned} \tag{7}$$

where

$$\begin{aligned} u_0 &= \cos(v\xi); \quad v_0 = \sin(v\xi); \quad v_0(t) = v_0 e^{-ik_y t l^2} \\ u_1 &= \cos(v\eta); \quad v_1 = \sin(v\eta); \quad v_1(t) = v_1 e^{-ik_y t l^2} \end{aligned} \tag{8}$$

The inverse transformations look as follows

$$a_p = u_0\alpha_p + v_0\left(p - \frac{k_x}{2}\right)\beta^{\dagger}_{k_x-p}$$

$$b_p = u_0\beta_p - v_0\left(\frac{k_x}{2} - p\right)\alpha^{\dagger}_{k_x-p}$$
$$c_p = u_1\gamma_p + v_1\left(p - \frac{k_x}{2}\right)\delta^{\dagger}_{k_x-p} \qquad (9)$$
$$d_p = u_1\delta_p - v_1\left(\frac{k_x}{2} - p\right)\gamma^{\dagger}_{k_x-p}$$

Now we will introduce the Hamiltonian of the two-dimensional electron-hole system describing the states of 2D e-h system in a strong perpendicular magnetic field taking into account two lowest Landau levels with $n_e, n_h = 0, 1$ for electrons and holes.

It can be written as follows

$$H = -\mu_{e0}\sum_p a^{\dagger}_p a_p - \mu_{h0}\sum_p b^{\dagger}_p b_p + \sum_p(\hbar\omega_{ce} - \mu_{e1})c^{\dagger}_p c_p + \\ + \sum_p(\hbar\omega_{ch} - \mu_{h1})d^{\dagger}_p d_p + \hat{H}^{(0,0)}_{Coul} + \hat{H}^{(1,1)}_{Coul} + \hat{H}^{(0,1)}_{Coul} + \hat{H}^{(1,0)}_{Coul}. \qquad (10)$$

The term $H^{(0,0)}_{Coul}$ describes the Coulomb interaction when the electrons and holes are situated on the LLLs with $n_e = n_h = 0$. The term $\hat{H}^{(1,1)}_{Coul}$ describes the Coulomb interaction between the particles situated on the first excited Landau levels $n_e = n_h = 1$, whereas the terms $\hat{H}^{(1,0)}_{Coul}$ and $\hat{H}^{(0,1)}_{Coul}$ describe Coulomb interactions when one particle is situated on the first level, for example, and the second partner of the interaction is situated on the lowest Landau level and vice versa. But here we took into account only the processes when the particles remained on the same Landau levels after the scattering. In this case, the number of particles on the given Landau level is conserved being constant. We also supposed that the quasi-equilibrium is established separately in the frame of the particles lying on the LLLs with $n_e = n_h = 0$ and apart for the particles occupying the FELLs $n_e = n_h = 1$. The scattering of the particles between two Landau levels with $n_e = 0$ and $n_e = 1$, for example, could be difficult in the absence of a needed optical phonon with the frequency ω_{L0} comparable with the electron cyclotron frequency ω_{ce}. By these reasons, we have introduced in Hamiltonian (10) two different chemical potentials $\mu_{e0} + \mu_{h0} = \mu_0$ and $\mu_{e1} + \mu_{h1} - \hbar\omega_{ce} - \hbar\omega_{ch} = \mu_1$.

The Hamiltonians describing four types of Coulomb interactions are the following

$$\hat{H}^{(0,0)}_{Coul} = \frac{1}{2}\sum_{p,q,s} F_{e-e}(p,0;q,0;p-s,0;q+s,0)a^{\dagger}_p a^{\dagger}_q a_{q+s} a_{p-s} + \\ + \frac{1}{2}\sum_{p,q,s} F_{h-h}(p,0;q,0;p-s,0;q+s,0)b^{\dagger}_p b^{\dagger}_q b_{q+s} b_{p-s} - \\ - \sum_{p,q,s} F_{e-h}(p,0;q,0;p-s,0;q+s,0)a^{\dagger}_p b^{\dagger}_q b_{q+s} a_{p-s};$$

$$\hat{H}^{(1,1)}_{Coul} = \frac{1}{2}\sum_{p,q,s} F_{e-e}(p,1;q,1;p-s,1;q+s,1)c^{\dagger}_p c^{\dagger}_q c_{q+s} c_{p-s} + \\ + \frac{1}{2}\sum_{p,q,s} F_{h-h}(p,1;q,1;p-s,1;q+s,1)d^{\dagger}_p d^{\dagger}_q d_{q+s} d_{p-s} - \\ - \sum_{p,q,s} F_{e-h}(p,1;q,1;p-s,1;q+s,1)c^{\dagger}_p d^{\dagger}_q d_{q+s} c_{p-s};$$

$$\begin{aligned}
\hat{H}^{(1,0)}_{Coul} &= \frac{1}{2}\sum_{p,q,s} F_{e-e}(p,1;q,0;p-s,1;q+s,0)c^{\dagger}_{p}a^{\dagger}_{q}a_{q+s}c_{p-s} + \\
&+\frac{1}{2}\sum_{p,q,s} F_{h-h}(p,1;q,0;p-s,1;q+s,0)d^{\dagger}_{p}b^{\dagger}_{q}b_{q+s}d_{p-s} - \\
&-\sum_{p,q,s} F_{e-h}(p,1;q,0;p-s,1;q+s,0)c^{\dagger}_{p}b^{\dagger}_{q}b_{q+s}c_{p-s}; \\
\hat{H}^{(0,1)}_{Coul} &= \frac{1}{2}\sum_{p,q,s} F_{e-e}(p,0;q,1;p-s,0;q+s,1)a^{\dagger}_{p}c^{\dagger}_{q}c_{q+s}a_{p-s} + \\
&+\frac{1}{2}\sum_{p,q,s} F_{h-h}(p,0;q,1;p-s,0;q+s,1)b^{\dagger}_{p}d^{\dagger}_{q}d_{q+s}b_{p-s} - \\
&-\sum_{p,q,s} F_{e-h}(p,0;q,1;p-s,0;q+s,1)a^{\dagger}_{p}d^{\dagger}_{q}d_{q+s}a_{p-s}.
\end{aligned} \tag{11}$$

The wave function of a single magnetoexciton in a superposition state can be obtained acting by operator (1) on the vacuum state $|0\rangle$ of the bare electron-hole operators

$$\left|\Psi_{ex}(k)\right\rangle = \hat{X}^{\dagger}(k)\left|0\right\rangle, \quad a_p\left|0\right\rangle = b_p\left|0\right\rangle = c_p\left|0\right\rangle = d_p\left|0\right\rangle = 0. \tag{12}$$

The ground state wave function of the Bose-Einstein condensed magnetoexcitons on the single particle state (12) is determined as

$$\left|\Psi_g(\vec{k})\right\rangle = \hat{D}\left(\sqrt{N_{ex}}\right)\left|0\right\rangle; \quad \left\langle\Psi_g(\vec{k})\right| = \left\langle 0\right|\hat{D}^{\dagger}\left(\sqrt{N_{ex}}\right). \tag{13}$$

It plays the role of the vacuum state for the transformed new operators α_p, β_p, γ_p, δ_p. This statement can be proved as follows

$$\alpha_p\left|\Psi_g(k)\right\rangle = Da_pD^{\dagger}D\left|0\right\rangle = Da_p\left|0\right\rangle = 0 \tag{14}$$

and so on. It means that

$$\begin{gathered}
\beta_p\left|\Psi_g(k)\right\rangle = \gamma_p\left|\Psi_g(k)\right\rangle = \delta_p\left|\Psi_g(k)\right\rangle = 0 \\
\left\langle\Psi_g(k)\right|\alpha^{\dagger}_p = \left\langle\Psi_g(k)\right|\beta^{\dagger}_p = \left\langle\Psi_g(k)\right|\gamma^{\dagger}_p = \left\langle\Psi_g(k)\right|\delta^{\dagger}_p = 0
\end{gathered} \tag{15}$$

The operators of the electron and hole numbers are

$$\hat{N}_e = \sum_p a^{\dagger}_p a_p + \sum_p c^{\dagger}_p c_p; \quad \hat{N}_h = \sum_p b^{\dagger}_p b_p + \sum_p d^{\dagger}_p d_p. \tag{16}$$

Their average numbers in the ground state (13) of the Bose-Einstein condensed magnetoexcitons equals to the full number of excitons $N_{ex} = v^2 N$

$$\left\langle\Psi_g\left|\hat{N}_e\right|\Psi_g\right\rangle = \left\langle\Psi_g\left|\hat{N}_h\right|\Psi_g\right\rangle = N_{ex} = Nv^2. \tag{17}$$

Equation (7) can be transcribed in the form

$$v^2 = \sin^2(v\xi) + \sin^2(v\eta); \quad \xi^2 + \eta^2 = 1. \tag{18}$$

The strong solution of this equation is the value $v = 0$, but it can be satisfied approximately by the small values $v^2 < 1$. It reflects the properties of the exciton creation and annihilation operators, which strictly speaking obey the Bose commutation relations only in the case $v = 0$, but can be considered as approximate bosons in the case $v^2 < 1$.

The unitary transformation $\hat{D}\left(\sqrt{N_{ex}}\right)$ (2) introducing the coherent macroscopic states was constructed using the approximate Bose operators instead of true ones, and this fact leads to requirement (18), which can be satisfied only approximately in the case $v^2 < 1$.

These relations permit us to determine the admissible values v^2, at which the BEC on the superposition state can take place. For example, any values $v^2 < 1$, when $\sin^2(v\xi) \approx v^2\xi^2$ and $\sin^2(v\eta) \approx v^2\eta^2$, satisfy these conditions. In the cases $\xi = 1$, $\eta = 0$ and $\xi = 0$, $\eta = 1$ they lead to earlier established relation in Ref. [5] $v^2 = \sin^2 v$, equivalent to the requirement $v^2 < 1$. In the case $\xi = \eta = 1/\sqrt{2}$ one can write

$$\frac{v^2}{2} = \sin^2\left(\frac{v}{\sqrt{2}}\right), \text{ i.e., } v^2 < 2. \tag{19}$$

Side by side with the parameter v^2, which plays the role of the main filling factor, there are also two partial filling factors $\sin^2(\xi v)$ and $\sin^2(\eta v)$, which reflect the phase space filling (PSF) effect of two phase spaces, one of them being related with the LLLs ($n_e = n_h = 0$) and the other one with FELLs ($n_e = n_h = 1$). Two phase spaces are engaged simultaneously because the superposition state $\hat{X}^\dagger(k)|0\rangle$ is composed by two bare excitons states $\hat{X}^{(0,0)\dagger}(k)|0\rangle$ and $\hat{X}^{(1,1)\dagger}(k)|0\rangle$ with the weights ξ^2 and η^2, correspondingly. Due to the exclusion Pauli principle, the creation of many excitons in the state $\hat{X}^{(0,0)\dagger}(k)|0\rangle$ means the occupations of the same number of electron states in the LLL with $n_e = 0$ and of the same number of holes states in the LLL with $n_h = 0$. In spite of the fact that the exciton binding energy is situated on the energy scale lower in comparison with the energy of a free electron-hole pair, the equal numbers of electron and hole states remain occupied during the BEC of excitons. In the case of superposition exciton state, two phase spaces corresponding to two sets of Landau levels in the process of the Bose-Einstein condensation are engaged. The process of BEC means in this case the simultaneous filling of both phase spaces in the proportions $\sin^2(\xi v)$ and $\sin^2(\eta v)$. Even when the lower energy states related with $\hat{X}^{(0,0)\dagger}(k)|0\rangle$ are not completely filled by the excitons, the electron and hole states related with $n_e = n_h = 1$ become partially filled. All these consequences of the Pauli exclusion principle are well known in the absence of a superposition state and are generalized in the presence of the superposition state. Since the fillings of the electron bands and of the phase spaces are determined by the corresponding chemical potentials, we can expect the necessity to introduce two independent chemical potentials μ_0 and μ_1 for two phase spaces mentioned above. Such a possibility is supported also by the considerations related with the electron-phonon and hole-phonon scattering processes between different Landau levels discussed in Ref. [5]. Now, using equations (7) we will introduce the new transformed operators α_p, β_p, γ_p and δ_p into the Hamiltonian. After its rewriting in a normal ordered form it will contain constant parts without operators, playing the role of the ground state energy, as well as the quadratic type Hamiltonian H_2 containing the diagonal terms of the types $\alpha_p^\dagger\alpha_p$, $\beta_p^\dagger\beta_p$, $\gamma_p^\dagger\gamma_p$, $\delta_p^\dagger\delta_p$ and nondiagonal terms of the types $\alpha_p^\dagger\beta_{k_x-p}^\dagger$, $\beta_{k_x-p}\alpha_p$, $\gamma_p^\dagger\delta_{k_x-p}^\dagger$ and $\delta_{k_x-p}\gamma_p$.

Hamiltonian (10) can be divided in three parts: $H = H(0) + H(1) + H(0,1)$. One of them $H(0)$ contains only the electron and hole operators $a_p^\dagger$, a_p, $b_p^\dagger$, b_p corresponding to the LLLs. It can be transformed using the partial unitary transformation $\hat{D}_0\left(\sqrt{N_{ex}}\right)$. The second part $H(1)$ contains only the electron and hole operators $c_p^\dagger$, c_p, $d_p^\dagger$, d_p corresponding to the FELLs. It can be transformed using the partial unitary transformation $\hat{D}_1\left(\sqrt{N_{ex}}\right)$. The third part $H(0,1)$ consists of two Hamiltonians of Coulomb interaction $H_{Coul}^{(1,0)}$ and $H_{Coul}^{(0,1)}$. For simplicity we will consider the electron-hole system with equal masses $m_e = m_h$, equal cyclotron frequencies $\omega_{ce} = \omega_{ch}$ and equal chemical potentials

$$\mu_{e0} = \mu_{h0} = \mu_0/2\,, \qquad (\mu_{e1} - \hbar\omega_{ce}) = (\mu_{h1} - \hbar\omega_{ch}) = \mu_1/2\,. \tag{20}$$

The transformation of the Hamiltonian $H(0)$ with the partial unitary transformation $\hat{D}_0\left(\sqrt{N_{ex}}\right)$ coincides exactly with the case considered in Ref. [5] and is expressed by formulas (39)-(43) [5]. We intent to use these results applying them to the Hamiltonian $H(0)$. By this reason, in formulas (39)-(43) of Ref. [5], we will add the supplementary label zero, as follows

$$\begin{gathered} H(0) \cong U(0) + H_2(0); \\ U(0) = -Nv_0^2\left[I_{ex}^{(0,0)}(k) + \mu_0\right] - Nv_0^4\left[I_l^{(0,0)} - I_{ex}^{(0,0)}(k)\right]; \\ H_2(0) = \sum_p E(0,k,v_0^2,\mu_0)\left(\alpha_p^\dagger\alpha_p + \beta_p^\dagger\beta_p\right) - \\ -\sum_p \Psi(0,k,v_0^2,\mu_0)\left[u_0 v_0\left(\frac{k_x}{2} - p\right)\beta_{k_x-p}\alpha_p + u_0 v_0\left(p - \frac{k_x}{2}\right)\alpha_p^\dagger\beta_{k_x-p}^\dagger\right]. \end{gathered} \tag{21}$$

One can remember that the dangerous diagrams describing the spontaneous appearance from the vacuum of the quasiparticle pairs were compensated in [5] by the condition $\Psi(0,k,v_0^2,\mu_0) = 0$, which permitted determining the unknown parameter of the system, namely, the chemical potential μ_0. It was shown that the chemical potential μ_0 and the single particle elementary excitation energy $E(0,k,v_0^2,\mu_0)$ in the Hartree-Fock-Bogoliubov approximation equal to

$$\mu_0 = -I_{ex}^{(0,0)}(k) - 2v_0^2\left[I_l^{(0,0)} - I_{ex}^{(0,0)}(k)\right]; \quad E(0,k,v_0^2,\mu_0) = \frac{I_{ex}^{(0,0)}(k)}{2}. \tag{22}$$

This result being applied to the subsystem with the Hamiltonian $H(0)$ and with the subspace created by the states with $n_e = n_h = 0$ in Landau gauge demonstrates the trends to collapse. It occurs because in the case of the magnetoexciton $\hat{X}^{(0,0)\dagger}(k)\left|0\right\rangle$ the difference $\left[I_l^{(0,0)} - I_{ex}^{(0,0)}(k)\right] \ge 0$ is positive and the chemical potential μ_0 decreases when the partial filling factor v_0^2 increases. In the case of Ref. [5], these results have determined completely the quantum statistical state of the system in HFBA. In our case, these results concern only the first subsystem. They can be considered as a partial, incomplete information, which must be supplemented by similar analysis of the second subsystem. Now we will perform the same procedure with the Hamiltonian $H(1)$ using the unitary transformation $\hat{D}_1\left(\sqrt{N_{ex}}\right)$.

The results will be similar with the previous ones, but now we must substitute μ_0 by μ_1, $I_{ex}^{(0,0)}(k)$ by $I_{ex}^{(1,1)}(k)$, $I_l^{(0,0)}$ by $I_l^{(1,1)}$, u_0 by u_1 and v_0 by v_1. We obtained

$$H(1) \cong U(1) + H_2(1)\,,$$

where

$$U(1) = -Nv_1^2\left[I_{ex}^{(1,1)}(k) + \mu_1\right] - Nv_1^4\left[I_l^{(1,1)} - I_{ex}^{(1,1)}(k)\right] \tag{23}$$

and

$$\begin{gathered} H_2(1) = \sum_p E(1, k, v_1^2, \mu_1)\left(\gamma_p^\dagger\gamma_p + \delta_p^\dagger\delta_p\right) - \\ -\sum_p \Psi(1, k, v_1^2, \mu_1)\left[u_1v_1\left(\frac{k_x}{2} - p\right)\delta_{k_x-p}\gamma_p + u_1v_1\left(p - \frac{k_x}{2}\right)\gamma_p^\dagger\delta_{k_x-p}^\dagger\right]. \end{gathered} \tag{24}$$

Here $E(1, k, v_1^2, \mu_1)$ and $\Psi(1, k, v_1^2, \mu_1)$ are

$$\begin{gathered} E(1, k, v_1^2, \mu_1) = 2u_1^2v_1^2I_{ex}^{(1,1)}(k) + I_l^{(1,1)}(v_1^4 - u_1^2v_1^2) - \frac{\mu_1}{2}(u_1^2 - v_1^2) \\ \Psi(1, k, v_1^2, \mu_1) = 2v_1^2I_l^{(1,1)} + I_{ex}^{(1,1)}(k)(u_1^2 - v_1^2) + \mu_1 \end{gathered} \tag{25}$$

The Hamiltonian $H_2(1)$ also contains dangerous terms describing the creation and annihilation of a second type pairs of new quasiparticles with total momentum k_x. The new vacuum state $\left|\Psi_g(k)\right\rangle$ will remain stable if the new compensation condition

$$\Psi(1, k, v_1^2, \mu_1) = 0 \tag{26}$$

takes place. It can be satisfied choosing the value of the new independent chemical potential μ_1. In the HFB approximation it equals to

$$\mu_1^{HFB} = -I_{ex}^{(1,1)}(k) - 2v_1^2\left[I_l^{(1,1)} - I_{ex}^{(1,1)}(k)\right] \tag{27}$$

and leads to the energy spectrum for a second type single particle excitation from the ground state $\left|\Psi_g(k)\right\rangle$

$$E(1, k, v_1^2, \mu_1) = \frac{I_{ex}^{(1,1)}(k)}{2}. \tag{28}$$

In the case of the first subsystem, the energy (22) of the single particle elementary excitation equals to one half of the ionization potential $I_{ex}^{(0,0)}(k)$ of the magnetoexciton in the bare state $\hat{X}^{(0,0)\dagger}(k)\left|0\right\rangle$. In the case of the second subsystem, the second type single particle elementary excitations have the energy $I_{ex}^{(1,1)}(k)/2$ equal to one half of the ionization potential $I_{ex}^{(1,1)}(k)$ of the magnetoexciton in a bare state $\hat{X}^{(1,1)\dagger}(k)\left|0\right\rangle$.

There are four types of single-particle elementary excitations represented by the new quasiparticle operators $\alpha_p^\dagger, \alpha_p$; $\beta_p^\dagger, \beta_p$; $\gamma_p^\dagger, \gamma_p$ and $\delta_p^\dagger, \delta_p$. There are two types of quasi-electrons and two types of quasi-holes, and their excitation energies (22) and (28) are determined independently by two different chemical potentials μ_0 and μ_1 as in the case of two independent Bose-Einstein condensates. The only interconnection between them is expressed by the partial filling factors $v_0^2 = \sin^2\xi v$ and $v_1^2 = \sin^2\eta v$, which obey requirement (18). They both depend on the filling factor v^2 and determine the phase space fillings of two subspaces: one of them being formed by the states related with the LLLs and the other being formed by the states related with FELLs. The existence of a possibility of two independent Bose-Einstein condensates arising simultaneously is determined by the structure of the unitary transformation $D\left(\sqrt{N_{ex}}\right)$ and its factorized form $\hat{D} = \hat{D}_0\hat{D}_1$. This possibility can be realized when two independent chemical potentials μ_0 and μ_1 may be introduced.

The above described picture remains valid, if the third part $H(0,1)$ of the full Hamiltonian does not influence it. The influence on the compensation conditions can arise only from the diagonal terms with $s=0$ of the Hamiltonian $H(0,1)$. Only the diagonal terms with $s=0$ from the Coulomb inter-Landau levels interactions could influence the compensation conditions changing the coefficients $\Psi(0,k,v_0^2,\mu_0)$ and $\Psi(1,k,v_1^2,\mu_1)$. But in the case $s=0$ the Coulomb matrix elements of the electron- electron, hole-hole, and electron-hole interactions coincide.

$$F_{e-e}(p,0;-q,1;p,0;-q,1)=F_{e-h}(p,0;q,1;p,0;q,1)=F(p,0;q,1;p,0;q,1). \qquad (29)$$

It permits us to transcribe this part of the Hamiltonian as follows

$$\begin{aligned} H(0,1)\big|_{diag} = \sum_{p,q} F(p,0;q,1;p,0;q,1)\Big[c_p^{\dagger}c_p \left(a_q^{\dagger}a_q - b_{-q}^{\dagger}b_{-q}\right) - \\ - d_p^{\dagger}d_p\left(a_{-q}^{\dagger}a_{-q} - b_q^{\dagger}b_q\right)\Big] \end{aligned} \qquad (30)$$

This expression after the u_0, v_0 and u_1, v_1 transformations does not contain free and quadratic terms at all, which could influence the partial ground state energies $U(0)$ and $U(1)$ as well as the quadratic Hamiltonians $H_2(0)$ and $H_2(1)$. There are only the quaternion terms of the types $u_0^2u_1^2\alpha_p^{\dagger}\alpha_p\gamma_p^{\dagger}\gamma_p$, $\beta_{k_x-p}\alpha_p\gamma_p^{\dagger}\delta_{k_x-p}^{\dagger}u_0u_1v_0v_1$, $u_0u_1v_0(p-k_x/2)v_1(p-k_x/2)\times$ $\times\alpha_p^{\dagger}\beta_{k_x-p}^{\dagger}\gamma_p^{\dagger}\delta_{k_x-p}^{\dagger}$ and so on, which are beyond the quadratic approximation and do not influence the obtained values of the chemical potentials μ_0 and μ_1, as well as the energies of the single particle elementary excitations. Up till now we have considered the BEC on the superposition state equivalent to simultaneous BEC in two complementary subspaces. They are related with two different groups of Landau levels, when one subspace is formed from the LLLs and their quantum states, whereas the second subspace is formed by the states belonging to FELLs. In both subspaces, the breaking of the gauge symmetries takes place and the coherent pairing between the electron-hole partners belonging to the same subspaces occurs.

Side by side with this case, one can consider another variant when in the lower energy subspace related with LLL and described by the Hamiltonian $H(0)$ the electron-hole liquid (EHL) happens to be formed, whereas in the higher energy subspace related with FELLs and described by the Hamiltonian $H(1)$ as earlier the BEC occurs. In this case, all results related with the BEC on the magnetoexciton state $\hat{X}^{(1,1)\dagger}(k)|0\rangle$ remain valid. The EHL involving the electrons and holes situated on the LLLs was considered in Ref. [5], and these results can be completely used here. As in the case of two simultaneous Bose-Einstein condensates, in the last case of simultaneous EHL and BEC the influence of the Hamiltonian $H(0,1)$ vanishes.

3. Bose-Einstein Condensation of two-dimensional magnetoexcitons on the excited Landau levels

Now the BEC of magnetoexcitons formed by the electrons on the first excited Landau level $n_e=1$ and by holes on the LLL with $n_h=0$ will be considered. We suppose that the electrons are injected on the LL $n_e=1$ and the holes with the same concentration are injected on the level $n_h=0$. The Hamiltonian describing such a system has the form

$$\begin{aligned}H=&\sum_p\left(E_g+\frac{3}{2}\hbar\omega_{ce}-\mu_e\right)c_p^\dagger c_p+\sum_p\left(\frac{1}{2}\hbar\omega_{ch}-\mu_h\right)b_p^\dagger b_p+\\&+\frac{1}{2}\sum_{p,q,s}F_{e-e}(p,1;q,1;p-s,1;q+s,1)c_p^\dagger c_q^\dagger c_{q+s}c_{p-s}+\\&+\frac{1}{2}\sum_{p,q,s}F_{h-h}(p,0;q,0;p-s,0;q+s,0)b_p^\dagger b_q^\dagger b_{q+s}b_{p-s}-\\&-\sum_{p,q,s}F_{e-h}(p,1;q,0;p-s,1;q+s,0)c_p^\dagger b_q^\dagger b_{q+s}c_{p-s}\end{aligned}\tag{31}$$

Instead of chemical potentials μ_e and μ_h, it is useful to introduce the notations

$$\bar{\mu}_e=\mu_e-E_g-{}^3\!/_2\hbar\omega_{ce};\quad \bar{\mu}_h=\mu_h-{}^1\!/_2\hbar\omega_{ch}.\tag{32}$$

Above the matrix elements of the Coulomb interaction were used

$$\begin{aligned}F_{e-e}(p,1;q,1;p-s,1;q+s,1)&=\sum_\kappa W_{s,\kappa}\left[1-\frac{\left(s^2+\kappa^2\right)l^2}{2}\right]^2 f\left(\kappa(p-q-s)l^2\right)\\F_{h-h}(p,0;q,0;p-s,0;q+s,0)&=\sum_\kappa W_{s,\kappa}f\left(\kappa(p-q-s)l^2\right)\\F_{e-h}(p,1;q,0;p-s,1;q+s,0)&=\sum_\kappa W_{s,\kappa}\left[1-\frac{\left(s^2+\kappa^2\right)l^2}{2}\right]f\left(\kappa(p+q)l^2\right),\end{aligned}\tag{33}$$

where

$$W_{s,\kappa}=V_{s,\kappa}\exp\left[\frac{-\left(s^2+\kappa^2\right)l^2}{2}\right];\qquad V_{s,\kappa}=\frac{e^2 2\pi}{\varepsilon\sqrt{s^2+\kappa^2}S};\qquad f(\kappa t)=\exp(i\kappa t l^2)\tag{34}$$

The exciton creation operator $\hat{X}^{(1,0)\dagger}(k)$ has the form

$$\hat{X}^{(1,0)\dagger}(k)=\frac{1}{\sqrt{N}}\sum_t e^{-ik_y t l^2}c^\dagger_{t+\frac{k_x}{2}}b^\dagger_{-t+\frac{k_x}{2}}.\tag{35}$$

As earlier, we are introducing the unitary transformation $\hat{D}\left(\sqrt{N_{ex}}\right)$ as well as the *u-v* transformation (7) from the initial operators c_p, b_p to the new operators γ_p, β_p, taking into account only the terms U and H_2. In terms of the made calculations one meets with the values

$$\sum_q F_{e-e}(p,n;q,n;p,n;q,n)=NW_{0,0};\qquad n=0,1.\tag{36}$$

$$\begin{aligned}&\sum_s F_{e-e}(p,1;p-s,1;p-s,1;p,1)=\sum_{\vec{Q}}W_{\vec{Q}}\left(1-\frac{Q^2l^2}{2}\right)^2=I_l^{(1,1)};\\&\sum_s F_{e-e}(p,0;p-s,0;p-s,0;p,0)=\sum_{\vec{Q}}W_{\vec{Q}}=I_l^{(0,0)};\\&\sum_s F_{e-h}(p,1;k_x-p,0;p-s,1;k_x-p+s,0)e^{ik_y s l^2}=\\&=\sum_{\vec{Q}}W_{\vec{Q}}\left(1-\frac{Q^2l^2}{2}\right)\exp\left(\frac{i\left[\vec{Q}\times\vec{k}\right]}{2}\right)=I_{ex}^{(1,0)}(\vec{k}).\end{aligned}\tag{37}$$

As a result of the made transformation, we have obtained

$$DHD^{\dagger} \approx U + H_2\,, \tag{38}$$

where

$$U = -(\bar{\mu}_e + \bar{\mu}_h)Nv^2 - \frac{v^4}{2}N\left(I_l^{(1,1)} + I_l^{(0,0)}\right) - u^2v^2 I_{ex}^{(1,0)}(\vec{k}) \tag{39}$$

and

$$\begin{aligned} H_2 = &\left[v^4 I_l^{(0,0)} - u^2v^2 I_l^{(1,1)} + 2u^2v^2 I_{ex}^{(1,0)}(\vec{k}) + (\bar{\mu}_h v^2 - \bar{\mu}_e u^2)\right]\sum_p \gamma_p^{\dagger}\gamma_p + \\ &+\left[v^4 I_l^{(1,1)} - u^2v^2 I_l^{(0,0)} + 2u^2v^2 I_{ex}^{(1,0)}(\vec{k}) + (\bar{\mu}_e v^2 - \bar{\mu}_h u^2)\right]\sum_p \beta_p^{\dagger}\beta_p + \\ &+u\left[-v^2\left(I_l^{(1,1)} + I_l^{(0,0)}\right) + \left(v^2 - u^2\right)I_{ex}^{(1,0)}(\vec{k}) - (\bar{\mu}_e + \bar{\mu}_h)\right]\times \\ &\times\sum_p\left[v\left(p - \frac{k_x}{2}\right)\gamma_p^{\dagger}\beta_{k_x-p}^{\dagger} + v\left(\frac{k_x}{2} - p\right)\beta_{k_x-p}\gamma_p\right]. \end{aligned} \tag{40}$$

Compensation condition of the dangerous diagrams, which cancels the coefficient standing before the sum index, permits us to determine the summary chemical potential $\bar{\mu} = \bar{\mu}_e + \bar{\mu}_h$

$$\bar{\mu} = -I_{ex}^{(1,0)}(k) - 2v^2\left[\frac{I_l^{(0,0)} + I_l^{(1,1)}}{2} - I_{ex}^{(1,0)}(k)\right]. \tag{41}$$

Supposing that $\bar{\mu}_e = \bar{\mu}_h = \bar{\mu}/2$ we can transcribe the quadratic Hamiltonian H_2 as follows

$$H_2 = \sum_p T_e(k)\gamma_p^{\dagger}\gamma_p + \sum_p T_h(k)\beta_p^{\dagger}\beta_p \tag{42}$$

with the energies $T_e(k)$ and $T_h(k)$ of the single particle elementary excitations

$$\begin{aligned} T_e(k) &= \frac{I_{ex}^{(1,0)}(k)}{2} + \frac{v^2}{2}\left(I_l^{(0,0)} - I_l^{(1,1)}\right) \\ T_h(k) &= \frac{I_{ex}^{(1,0)}(k)}{2} - \frac{v^2}{2}\left(I_l^{(0,0)} - I_l^{(1,1)}\right). \end{aligned} \tag{43}$$

Here another source of instability appears because $T_h(k)$ can became negative at great values of k.

The total energy necessary to extract one e-h pair from the condensate equals exactly to the ionization potential of the magnetoexciton $X^{(1,0)}$, as it must be

$$T_e(k) + T_h(k) = I_{ex}^{(1,0)}(k)\,. \tag{44}$$

As a result, we must conclude that the BEC of the magnetoexcitons $X^{(1,0)}$ is unstable against the collapse because the values $I_{ex}^{(1,0)}(k)$ are less than the middle value $\left(I_l^{(0,0)} + I_l^{(1,1)}\right)/2$.

Above we have revised the possibilities of the BEC phenomenon to occur in the frame of the superposition state as well in the case of magnetoexcitons formed on the excited Landau levels. We can conclude that in the HFB approximation the BEC of magnetoexcitons on the LLLs remains a more representative and a simple model.

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