

# GENERATION OF THE NON-CLASSICAL STATES OF THE ELECTROMAGNETIC FIELD INTERACTING WITH A PAIR OF COLD ATOMS

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## Abstract

The interaction between the pair of cold two-level atoms and the single-mode cavity field is investigated. The two-level atoms in the pair are supposed to be indistinguishable. This problem generalizes the two-photon Jaynes-Cummings model of a single two-level atom interacting with the squeezed vacuum. The model of the pair of indistinguishable two-level atoms is equivalent to the problem of the equidistant three-level radiator with equal dipole moment matrix transition elements between the adjacent energy levels. Supposing that at the initial moment the field is in the squeezed vacuum state we obtain the exact analytical solution for the atom-field state-vector. By using this solution the quantum-statistical and squeezing properties of the radiation field are studied and compared with those for the single two-level atom system. It is observed that in the model of the pair of cold two-level atoms the exact periodicity of the squeezing revivals is violated by the analogy with the single two-level atom one.

## 1. Introduction

The subject of laser cooling and trapping of ions and neutral particles [1-2] at present is of great experimental and theoretical interest. It has been applied to quantum information processes [3-4], high-resolution spectroscopy, low-energy collisions, quantum jumps, photon antibunching [5], and new ultra-cold chemistry [6].

Since the laser cooling and trapping of the atomic pairs was demonstrated [7] it is important to investigate the generation of the non-classical states of the electromagnetic field in such systems. For example, the entanglement evolution of two two-level systems, coupled to  $N$  surrounding spin  $1/2$  was carefully studied in Ref. [8].

On the other hand, the squeezing revivals of the single-mode cavity field interacting with pair of cold atoms via intensity-dependent coupling were analyzed in detail in Ref. [9]. It was found that the exact periodicity of the squeezing oscillations in the two-atom system is violated, whereas in the single-atom one the exact periodicity of the squeezing revivals takes place [10]. In this situation it is important to generalize this model to the two-photon Jaynes-Cummings model of the pair of indistinguishable two-level atoms interacting with the single-mode cavity field. We are interested to compare the quantum-statistical properties of the radiation field in the two-atom model with the single-atom one [11].

As in the proposed model the two-level atoms are supposed to be indistinguishable, the quantum states of the pair of two-level atoms may be described in the three-level state representation  $|g\rangle$ ,  $|e_1\rangle$  and  $|e_2\rangle$  [9, 12, 13], where  $|g\rangle$ ,  $|e_1\rangle$  and  $|e_2\rangle$  are the ground, the first excited and the second excited atomic states, respectively. In other words, the pair of indistinguishable

two-level atoms is equivalent to the three-level equidistant radiator in the case in which the transitions between the two neighbouring Rydberg levels are resonantly coupled to a single mode of the cavity field. Since the squeezed light used in the present model is of the two-photon type we investigate the two-photon generalization [14, 15] of the Jaynes-Cummings model, whose interaction terms can be written through the SU(1,1) symmetry group generators.

## 2. The time-evolution of the state-vector of the atom-field coupled system

Let us investigate the interaction between the pair of indistinguishable two-level atoms (or equidistant three-level radiator with equal dipole moment matrix transition elements) and the single-mode cavity field. If we consider only two-photon transitions between the adjacent energy levels [14, 15] then our system can be described by the following Hamiltonian

$$H = \hbar\omega_0 S_z + \hbar\omega a^\dagger a + \hbar\lambda \{a^{+2} S^- + a^2 S^+\}. \quad (1)$$

Here

$$\begin{aligned} S^+ &= \sqrt{2}(|e_2\rangle\langle e_1| + |e_1\rangle\langle g|), \\ S^- &= \sqrt{2}(|e_1\rangle\langle e_2| + |g\rangle\langle e_1|) \end{aligned} \quad (2)$$

are the atomic raising and lowering operators, respectively,

$$S_z = |e_2\rangle\langle e_2| - |g\rangle\langle g| \quad (3)$$

is the atomic population inversion operator,  $\lambda$  is the atom-field coupling constant,  $\omega$  and  $\omega_0$  are the frequencies of the single-mode cavity field and the electronic transitions, respectively.

Let us represent Hamiltonian (1) through the SU(1,1) symmetry group generators

$$K^+ = \frac{a^{+2}}{2}, \quad K^- = \frac{a^2}{2}, \quad K_0 = \frac{a^\dagger a + a a^\dagger}{4} = \frac{1}{2}(N + \frac{1}{2}), \quad (4)$$

where  $N = a^\dagger a$  is the photon number operator. These operators satisfy the commutation relations for the SU(1,1) Lie algebra [9-11, 16-18]

$$[K_0, K^\pm] = \pm K^\pm, \quad [K^+, K^-] = -2K_0. \quad (5)$$

In this situation one can rewrite the Hamiltonian of the atom-field system by the following

$$H = \hbar\omega_0 S_z + 2\hbar\omega(K_0 - \frac{1}{2}) + 2\hbar\lambda \{K^+ S^- + K^- S^+\}. \quad (6)$$

The form of Hamiltonian (6) is similar with that for the Jaynes-Cummings model of the pair of atoms with the intensity-dependent coupling [9], whereas the realizations of the operators  $K^\pm$  and  $K_0$  in these two models are substantially different.

Let us find the time-evolution of the state-vector  $|\psi(t)\rangle$  of the coupled atom-field system. We assume that at the initial moment  $t=0$  the single-mode cavity field is in the squeezed vacuum state  $|\xi\rangle$  [11, 14, 19]

$$|\xi\rangle = (1 - |\xi|^2)^k \sum_{m=0}^{\infty} \left( \frac{\Gamma(m+2k)}{m! \Gamma(2k)} \right)^{1/2} \xi^m |m, k\rangle, \quad (7)$$

where the Bargmann index  $k=1/4$  corresponds to the standard oscillator realization (4),  $|m, k\rangle$  are the eigenvectors of the Casimir operator  $C^2 = K_0^2 - (K_+ K_- + K_- K_+)/2 = k(k-1)I$ ,  $\xi = |\xi| \exp(i\varphi)$  ( $0 \leq |\xi| \leq 1$ ). It should be emphasized that the squeezed vacuum state (7) is described by the Gaussian wave-function in comparison with the Holstein-Primakoff SU(1,1) coherent state [9, 10] for  $k=1/2$  the wave-function of which is not a Gaussian one.

If at the initial moment  $t = 0$  the three-level radiator is supposed to be in the first excited state  $|e_1\rangle$  for the initial state-vector  $|\psi(t=0)\rangle$  one can write

$$|\psi(t=0)\rangle = |e_1\rangle \otimes |\xi\rangle = \sum_{n=0}^{\infty} Q_n |n, e_1\rangle. \quad (8)$$

Here the coefficients  $Q_n$  are given by

$$Q_n = (1 - |\xi|^2)^{1/4} \left( \frac{\Gamma(n+1/2)}{n! \Gamma(1/2)} \right)^{1/2} \xi^n \quad (9)$$

and the states  $\{|n\rangle\}$  are not strictly number states but actually contain even numbers of photons  $2m$ , where  $m = 0, 1, 2, \dots$

The exact analytical solution for the atom-field state-vector one can obtain by solving the Schrödinger equation

$$i\hbar \frac{d|\psi(t)\rangle}{dt} = H|\psi(t)\rangle$$

in the resonance case  $\omega_0 = 2\omega$

$$\begin{aligned} |\psi(t)\rangle = & \sum_{n=0}^{\infty} e^{-i\omega_0(S_z + 2n)t} Q_n \left\{ \cos(\sqrt{n^2 + n + 1}\tau) |e_1, n\rangle \right. \\ & - i \frac{\sqrt{(n+1)(n+2)} \sin(\sqrt{n^2 + n + 1}\tau)}{\sqrt{2(n^2 + n + 1)}} |g, n+2\rangle \\ & \left. - i \frac{\sqrt{n(n-1)}}{\sqrt{2(n^2 + n + 1)}} \sin(\sqrt{n^2 + n + 1}\tau) |e_2, n-2\rangle \right\}, \end{aligned} \quad (10)$$

where  $\tau = 2\lambda t$  is the dimensionless time. In the next section by using this solution we examine the two-photon quantum-statistical properties of the single-mode cavity field, which initially is supposed to be in the squeezed vacuum state.

### 3. Two-photon quantum-statistical properties of the radiation field

Let us investigate the quantum-statistical properties of the single-mode cavity field interacting with the pair of cold atoms. From physical point of view it is important to compare these features with those for the single two-level atom model [11]. With the help of the analytical solution (10) one can easily obtain the following result for the atomic population inversion  $\langle S_z \rangle$

$$\langle S_z \rangle = - \sum_{n=0}^{\infty} |Q_n|^2 \frac{2n+1}{n^2 + n + 1} \sin^2(\sqrt{n^2 + n + 1}\tau). \quad (11)$$

In a similar way, we find the result for the mean photon number  $\langle n \rangle$  in the form

$$\begin{aligned} \langle n \rangle = & \sum_{n=0}^{\infty} |Q_n|^2 \cos^2(\sqrt{n^2 + n + 1}\tau) n \\ & + \sum_{n=0}^{\infty} |Q_n|^2 \{ (n+1)(n+2)^2 + n(n-1)(n-2) \} \frac{\sin^2(\sqrt{n^2 + n + 1}\tau)}{2(n^2 + n + 1)} \end{aligned} \quad (12)$$

In this situation both the atomic population inversion and the mean photon number have the tendency towards oscillations, but the exact periodicity of these oscillations is violated.

Let us consider the fluctuations in the mean photon number  $\sigma$

$$\sigma = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle}, \quad (13)$$

where  $\langle n^2 \rangle$  is given by the following

$$\begin{aligned} \langle n^2 \rangle = & \sum_{n=0}^{\infty} |Q_n|^2 \cos^2(\sqrt{n^2 + n + 1}\tau) n^2 \\ & + \sum_{n=0}^{\infty} |Q_n|^2 \{ (n+1)(n+2)^3 + n(n-1)(n-2)^2 \} \frac{\sin^2(\sqrt{n^2 + n + 1}\tau)}{2(n^2 + n + 1)} \end{aligned} \quad (14)$$

In the present model of the pair of indistinguishable two-level atoms the exact periodicity of the fluctuations  $\sigma$  is violated by the analogy with the micromaser one [20, 21] and the sub-Poissonian photon statistics periodically exhibits the quantum collapses and revivals.

It is important to study the generation of squeezed states of the quantized cavity field [22, 23], which initially is supposed to be in the squeezed vacuum state  $|\xi\rangle$ . For this purpose, we introduce the slowly varying operators  $A$  and  $A^\dagger$

$$A = ae^{i\omega t}, \quad A^\dagger = a^\dagger e^{-i\omega t}. \quad (15)$$

In this situation the real and imaginary parts of the single-mode field amplitude are given by

$$X_1 = \frac{A + A^\dagger}{2}, \quad X_2 = \frac{A - A^\dagger}{2i}. \quad (16)$$

These operators obey the commutation relation  $[X_1, X_2] = i/2$  and, as a result, they satisfy the uncertainty relation

$$\Delta X_1 \Delta X_2 \geq 1/4. \quad (17)$$

A state of the field is squeezed in the  $X$  variable if

$$\langle (\Delta X_i)^2 \rangle < 1/4, \quad i = 1, 2. \quad (18)$$

This condition may be rewritten in another form

$$-1 < S_i < 0, \quad (19)$$

where

$$S_i(t) = \frac{\langle (\Delta X_i)^2 \rangle - \langle (\Delta X_i)^2 \rangle_{coh}}{\langle (\Delta X_i)^2 \rangle_{coh}} \quad (20)$$

and  $\langle (\Delta X_i)^2 \rangle_{coh} = 1/4$ . It should be noted that such states are non-classical.

Let us calculate the function  $S_1$  numerically. Taking into account that this function is expressed through the mean values of the single-mode cavity field operators  $a$ ,  $a^2$ ,  $a^\dagger$  and  $a^{\dagger 2}$  one can obtain the following result

$$S_1(t) = 2[A_0(t) - A_2(t)] + 4\cos^2(\varphi)[A_2(t) - A_1^2(t)], \quad (21)$$

where  $A_0(t) = \langle n \rangle$ ,

$$\begin{aligned} A_1(t) = & |\xi| \sum_{n=0}^{\infty} P_n [\sqrt{n+1} \cos(\sqrt{n^2 + n + 1}\tau) \cos(\sqrt{n^2 + 3n + 3}\tau) \\ & + \{ (n+2)(n+3) + (n-1)n \} \sqrt{n+1} \\ & \times \frac{\sin(\sqrt{n^2 + n + 1}\tau) \sin(\sqrt{n^2 + 3n + 3}\tau)}{2\sqrt{n^2 + n + 1}\sqrt{n^2 + 3n + 3}}] \left[ \frac{\Gamma(n+1/2)\Gamma(n+3/2)}{n!(n+1)!\Gamma^2(1/2)} \right]^{1/2}, \end{aligned} \quad (22)$$

$$\begin{aligned}
 A_2(t) = & |\xi|^2 \sum_{n=0}^{\infty} P_n [\sqrt{(n+1)(n+2)} \cos(\sqrt{n^2+n+1}\tau) \cos(\sqrt{(n+2)^2+(n+2)+1}\tau) \\
 & + \{(n+3)(n+4) + (n-1)n\} \sqrt{(n+1)(n+2)} \\
 & \times \frac{\sin(\sqrt{n^2+n+1}\tau) \sin(\sqrt{(n+2)^2+(n+2)+1}\tau)}{2\sqrt{n^2+n+1}\sqrt{(n+2)^2+(n+2)+1}} \left[ \frac{\Gamma(n+1/2)\Gamma(n+5/2)}{n!(n+2)!\Gamma^2(1/2)} \right]^{1/2}, \\
 P_n = & |Q_n|^2 = (1-|\xi|^2)^{1/2} \frac{\Gamma(n+1/2)}{n!\Gamma(1/2)} |\xi|^{2n}.
 \end{aligned} \tag{23}$$

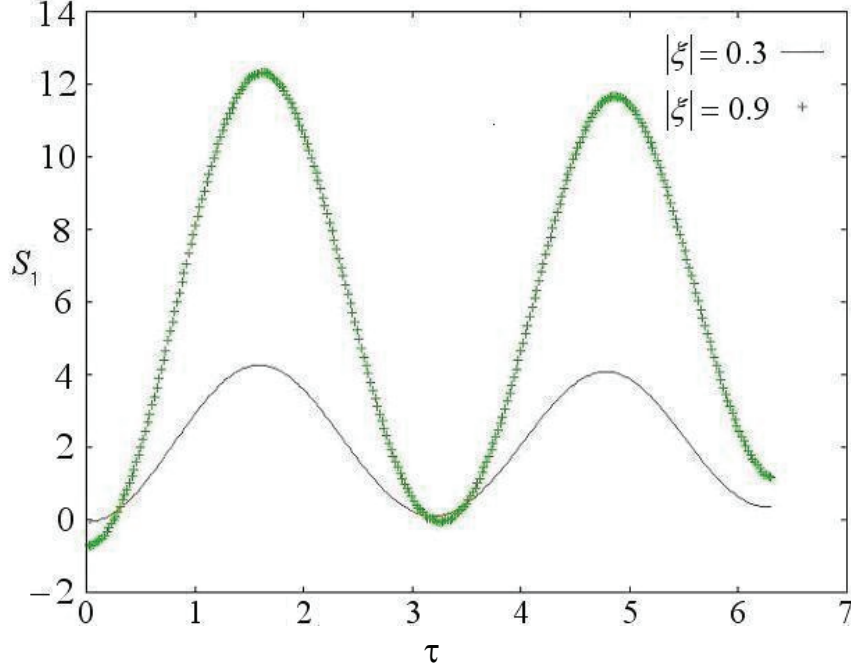


Fig. 1.  $S_1$  as a function of the dimensionless time  $\tau$  for  $|\xi|=0.3$  and  $0.9$ .

Figure 1 shows the function  $S_1$  against  $\tau$  for the different values  $|\xi|=0.3$  and  $0.9$ . It is observed that with the increasing of the initial mean photon number  $\langle n(t=0) \rangle = |\xi|^2 / (1 - |\xi|^2)$  the squeezing enhances. The main result consists in the fact that the exact periodicity of the squeezing revivals is violated in the two-photon model of the pair of indistinguishable two-level atoms by the analogy with the single atom one [11]. In this situation it should be noted that the exact squeezing periodicity is also violated in the interaction of the quantized electromagnetic field with the pair of cold atoms via intensity-dependent coupling [9].

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