EFFECTS OF VISCOUS AND JOULEAN DISSIPATIONS ON HYDRO-MAGNETIC FREE CONVECTION FLOW OVER A HEATED SURFACE

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In this paper, effects of viscous and Joulean dissipations on hydro-magnetic convection flow on an incompressible fluid with temperature dependent viscosity over a stretching surface have been studied numerically using MATLAB 6.1. The results for velocity and temperature distribution, local skin friction coefficient, and surface heat flux coefficients have been investigated and discussed for different values of the parameters encountered in the equations.

1. Introduction

Studies of transfer of heat, mass and momentum in the laminar boundary layer flow on a heated stretching sheet gained importance from both theoretical and practical points of view in many scientific and engineering applications, especially in polymer processing industry, liquid thin-film, and other related surface flows. Sakiadis [9] studied boundary layer flow over a continuous solid surface moving in its own plane with a constant speed. Erickson [1] have taken into account transverse non-zero velocity at the moving surface and heat and mass transfer in the boundary layer which are important where polymer sheets are extruded continuously from a die. Tsou, Sparrow and Goldstien [11] studied further the same problem without considering viscous dissipative heat which plays an important role in the rate of heat transfer. McCormack and Crane [6], Gupta and Gupta [2] also considered suction or blowing in the same investigation. Rawdan and Elbashbeshy [8] studied variable concentration in a transverse magnetic field for a conducting liquid. But in all the above cases the viscosity of the fluid was supposed to be uniform. Jang and Leu [4] first considered temperature dependent viscosity in their study. Recently Lahiri et al. [5] analyzed effects of transverse magnetic field in the momentum and heat transfer characterized in the boundary layer of an incompressible conducting fluid flow over a heated stretching sheet where viscosity of a fluid depends on temperature. However, in these studies the effect of free convection has not been considered either and the effects of viscous and Joulean dissipation on the flow have also been neglected.

In the present paper, we propose to investigate these effects on the free convection boundary layer flow of conducting fluid with variable viscosity over a heated stretching surface. Also we study the effects of viscosity variation parameter (ξ), Hartmann number (M), Eckert number (E), Grashoff number (G_r), Prandtl number (P_r), etc. on velocity distribution, temperature distribution, local skin friction, and surface heat flux coefficient numerically; their natures are shown graphically.

2. Mathematical formulations

Let us consider the steady two-dimensional free convection flow of a viscous incompressible, electrically conducting fluid over a heated stretching sheet. The motion of the fluid is being caused by the surface which is moving horizontally with a velocity proportional to the distance from the origin ($x = 0$). Viscosity of the fluid is assumed to be dependent on temperature, as in many practical situations this physical property may change with temperature. A physical model of the problem with co-ordinate system is shown in Fig. 1.

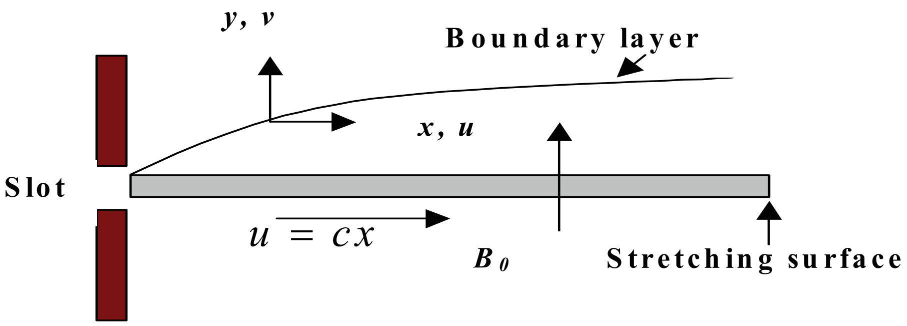


Fig. 1. The physical model of the stretching sheet and the coordinate system.

The governing equations of such a boundary layer flow in presence of a uniformly distributed transverse magnetic field of strength B_0 are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma {B_0}^2 u}{\rho}\,, \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma {B_0}^2 u^2\,, \tag{3}$$

where u, v are the components of velocity along the x and y directions, respectively; T, is the temperature of the fluid in the boundary layer; T_∞ is the free stream temperature; ρ is the density of the fluid; σ is the conductivity of the fluid; α is the coefficient of thermal diffusivity; β is the coefficient of thermal expansion; μ is the coefficient of fluid viscosity; C_p is the specific heat of the fluid under constant pressure, and g is the acceleration due to gravity.

The boundary conditions are

$$u = cx,\ v = 0,\quad T = T_w \text{ at } y = 0\,, \tag{4}$$

$$u \to 0,\ T \to T_\infty \text{ at } y \to \infty\,, \tag{5}$$

where T_w is the uniform wall temperature and c(>0) is constant.

We now introduce the following relations for u, v, and θ as

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \tag{6}$$

and

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{7}$$

where ψ is the stream function. The temperature dependent viscosity is given by (Bird et al. [1])

$$\mu = \mu_r^* e^{a(T_w - T)}, \tag{8}$$

where μ^* is the reference viscosity and a is a constant.

Using relations (6), (7) in (2) and (3) we get,

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = -\xi v^* e^{\xi(1-\theta)}\frac{\partial\theta}{\partial y}\frac{\partial^2\psi}{\partial y^2} + G_r c^2 x\theta + v^* e^{\xi(1-\theta)}\frac{\partial^3\psi}{\partial y^3} - Mc\frac{\partial\psi}{\partial y} \tag{9}$$

From the energy equation (3), we have

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y} = \alpha\frac{\partial^2\theta}{\partial y^2} + \frac{v^* E e^{\xi(1-\theta)}}{c^2x^2}\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 + \frac{ME}{cx^2}\left(\frac{\partial\psi}{\partial y}\right)^2, \tag{10}$$

where c has a dimension (1/time), $v^* = \dfrac{\mu^*}{\rho}$, $\alpha = \dfrac{k}{\rho C_p}$,

$$\xi = a(T_w - T_\infty) \tag{11}$$

is the viscosity variation parameter,

$$G_r = \frac{g\beta(T_w - T_\infty)}{c^2 x} \tag{12}$$

is the Grashoff Number,

$$E = \frac{c^2x^2}{C_p(T_w - T_\infty)} \tag{13}$$

is the Eckert number, and

$$M = \frac{\sigma B_0{}^2}{\rho c} \tag{14}$$

is the Hartmann number.

Further we apply the similarity transformations

$$\psi = (cv^*)^{1/2} x f(\eta), \qquad \eta = \left(\frac{c}{v^*}\right)^{1/2} y \tag{15}$$

to equations (9) and (10) and obtain

$$(f')^2 - ff'' + \xi e^{\xi(1-\theta)}\theta' f'' = e^{\xi(1-\theta)} f''' + G_r\,\theta - Mf' \tag{16}$$

and

$$\theta'' + P_r f\theta' + P_r E e^{\xi(1-\theta)}(f'')^2 + P_r EM(f')^2 = 0, \tag{17}$$

where

$$Pr = \frac{v}{\alpha} = \frac{\mu C_p}{k} \tag{18}$$

is the Prandtl number.

Boundary conditions (4) and (5) reduce to

$$f(0) = 0, \qquad f'(0) = 1, \qquad \theta(0) = 1 \quad \text{and} \tag{19}$$

$$f'(\infty) = 0, \qquad \theta(\infty) = 0. \tag{20}$$

In the above equations, the prime indicates the differentiation w.r.t. η.

3. Method of solution

The non-linear differential equations (16) and (17) with boundary conditions (19) and (20) have been solved in MATLAB 6.1 using finite difference code that implements the 3-stage Lobatto IIIa formula for partitioned Runge-Kutta method. This is a collocation formula and the collocation polynomial provides a C1-continuous solution that is fourth order accurate. Mesh selection and error control are based on the residual of the continuous solution. The system cannot be solved on an infinite interval, and it would be impractical to solve it even for a very large finite interval. So, we tried to solve a sequence of problems posed on increasingly larger intervals to verify the solution consistent behavior as the boundary approaches ∞. The plot of each successive solution is superimposed over those of previous solutions so that they can easily be compared for consistency. For numerical computations we take infinity condition at a large but finite value of η where no considerable variation in velocity and temperature occurs.

4. Numerical results and discussions

The velocity and temperature distributions are calculated for different values of viscosity variation parameter, Hartmann number, Eckert number, Grashoff number, and Prandtl number and these are depicted in Figs. 2 to 7.

For velocity, we plot $f'(\eta)$ against η and for temperature, we plot $\theta(\eta)$ against η.

It can be easily seen from Fig. 2 that the velocity decreases as η increases for a fixed value of G_r, but the rate of decrease is faster as G_r increases. For a nonzero fixed value of η, the velocity decreases when G_r increases. On the other hand, the temperature decreases with the increase of η, but the increase in the rate of change of temperature with the increase of G_r is not significant (Fig. 3).

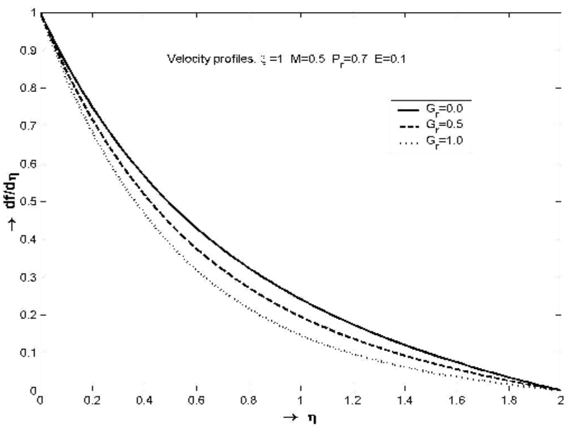


Fig. 2. Velocity profiles against η when G_r changes.

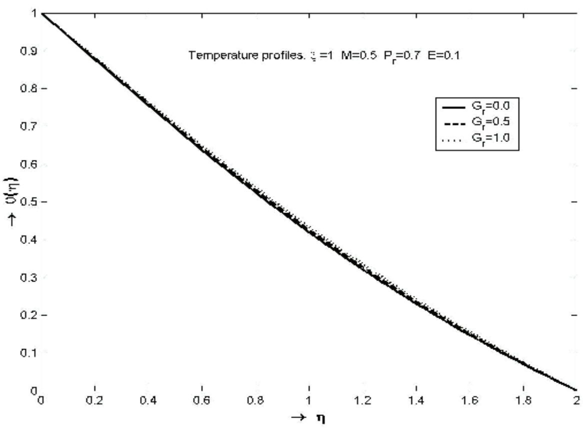


Fig. 3. Temperature profiles against η when G_r change.

It is seen from Fig. 4, that the decrease of velocity is slightly faster when the value of E increases, which is not significant. However, for a nonzero fixed value of η, the velocity decreases when E increases. On the other hand, Fig. 5 shows that the temperature decreases rapidly at first and then slowly with the increase of η and there is a significant increase in the rate of decrease of temperature with the increase of E.

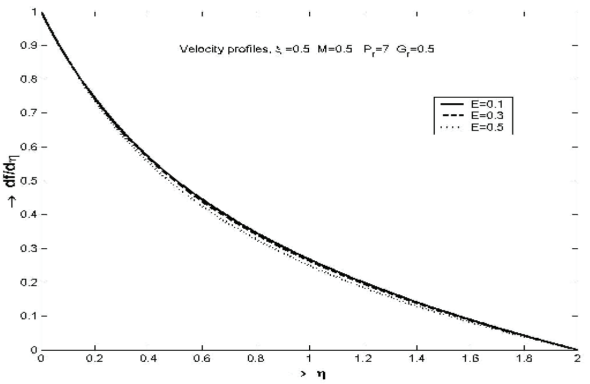


Fig. 4. Velocity profiles against η when E changes.

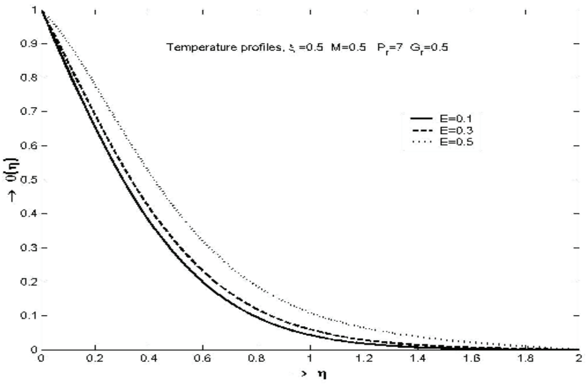


Fig. 5. Temperature profiles against η when E changes.

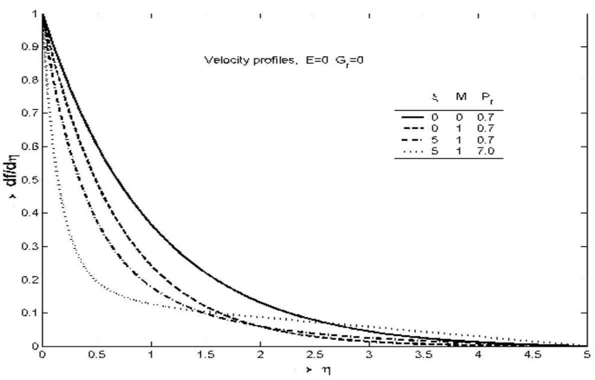


Fig. 6. Velocity profiles against η when G_r and E have no effect.

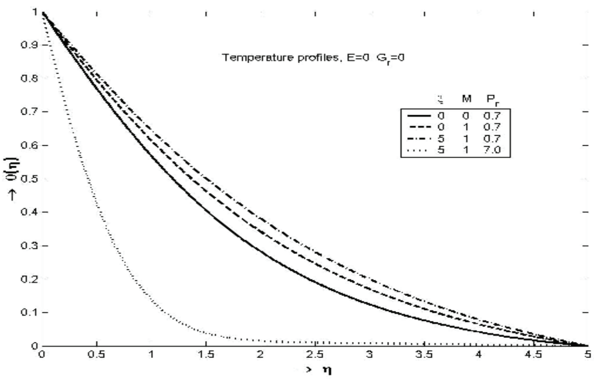


Fig. 7. Temperature profiles against η when G_r and E have no effect.

Figure 6 shows that the velocity decreases as η increases, but the rate of decrease of velocity significantly increases for any of the cases when ξ, M, or P_r increases for zero value of E and G_r which is consistent with the results analyzed by previous studies (Lahiri et al. [5]). For zero value of E and G_r, Fig. 7 shows that the temperature decreases as η increases. The rate of decrease in temperature is significant when ξ or M increases, but it significantly increases when P_r increases, which is also consistent with the results analyzed by previous studies.

Local skin friction: An important characteristic of the boundary layer flow is the local skin-friction coefficient and is given by $-f''(0)$. The effect of viscosity variation parameter, Hartmann number, Eckert number, and Prandtl number on local skin friction when Grashoff number varies are depicted graphically in Figs. 8 to 11.

It is noted from the figures that when G_r increases, the local skin friction coefficient increases, which is uniform when ξ or M increases for any fixed value of G_r (Figs. 8, 9), but the rate of increase slows down for large Prandtl number (Fig. 10). The local skin friction coefficient slightly decreases when the Eckert number increases for any fixed value of G_r (Fig. 11).

Surface heat flux coefficient: Another important characteristic of boundary layer flow over a heated surface is the surface heat flux coefficient and is given by $-\theta'(0)$. We investigate graphically the effect of viscosity variation parameter, Hartmann number, Eckert number, and Prandtl number when Grashoff number changes on the surface heat flux coefficient shown in Figs. 12 to 15.

Fig. 8.

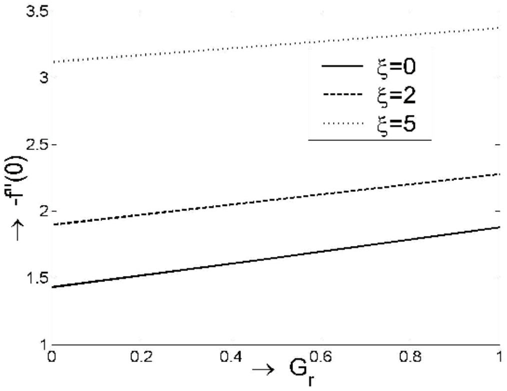


Fig. 9.

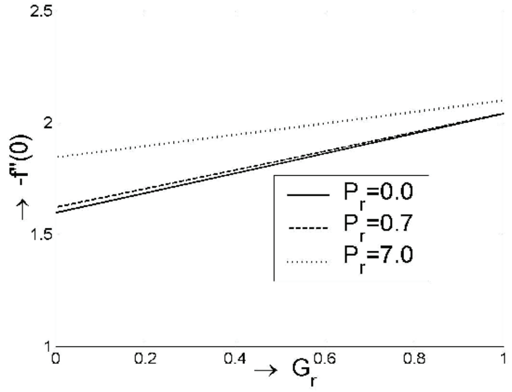


Fig. 10.

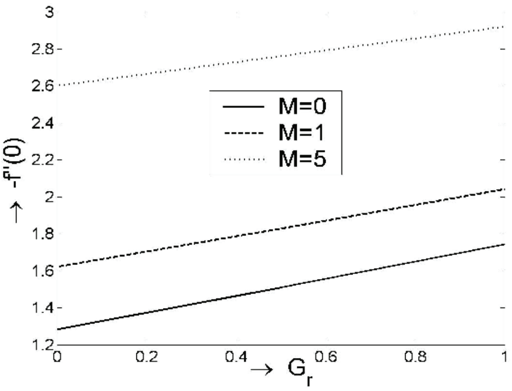
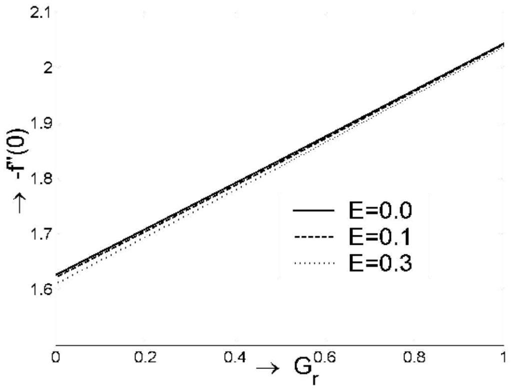


Fig. 11.



Change of local skin friction coefficient (Figs. 8-11).

Fig. 12.

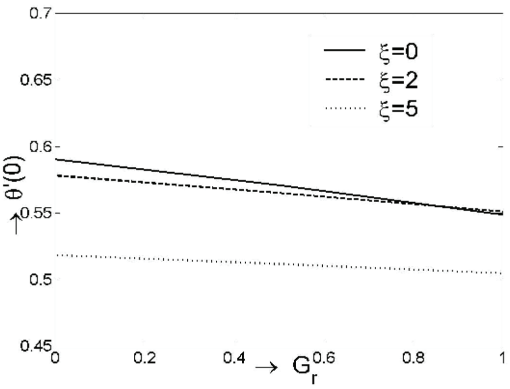


Fig. 13.

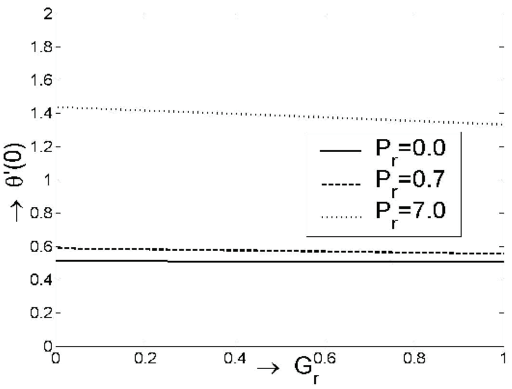


Fig. 14.

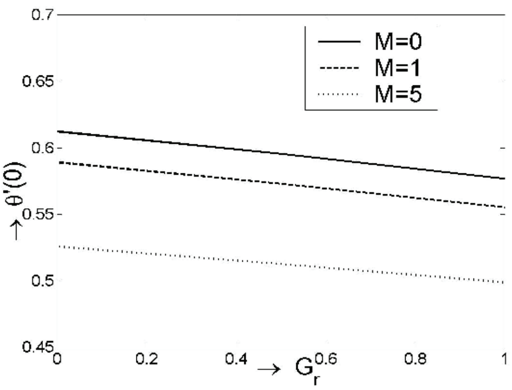
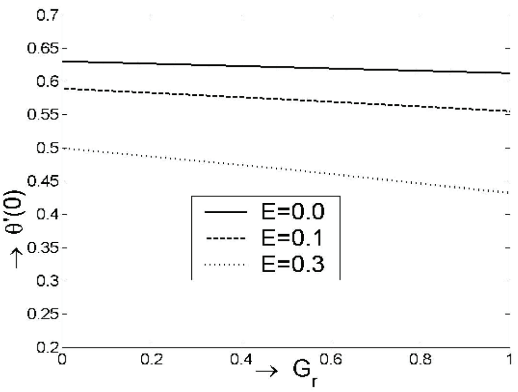


Fig. 15.



Change of surface heat flux coefficient (Figs. 12-15).

It is clear from the figures that when G_r increases, the surface heat flux coefficient slightly decreases, which is uniform when P_r, E or M increases for any fixed value of G_r (Figs. 13, 14, 15), but the rate of decrease slows down when viscosity variation parameter changes from zero to a non-zero value (Fig. 12). On the other hand, the surface heat flux coefficient increases when Prandtl number increases for any fixed value of G_r (Fig. 14).

5. Conclusions

As in nuclear engineering, convection aids the cooling of reactors and, therefore, the free convection boundary layer flow of an electrically conducting fluid over a heated stretching surface is much more important from the application point of view in industry. Finally, it may be suggested that the effect of the viscosity variation, magnetic field, viscous and Joulean dissipation, etc. may be investigated for surface flow of the other geometry, which is also of great interest in terms of application in polymer industry.

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