

PARTIAL REVERSIBILITY OF ATOMIC INVERSION IN OFF-RESONANCE INTERACTION OF RADIATOR WITH QUANTUM CAVITY FIELD

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(Received 19 March 2008)*

Abstract

The off-resonance effect and its influence on the reversibility between two quantum subsystems (single mode cavity field and a three-level in cascade configuration) in interaction are studied. The partial reversible condition is found for which these quantum subsystems can restore their diagonal moments, while the non-diagonals remain correlated after the interaction process.

1. Introduction

Actually, in quantum physics a considerable attention has been paid to the resonance interaction between one atom and the quantized field. This type of interaction was frequently used in classical physics for transmission or manipulation in time of information and remains attractive in quantum description of these effects [1, 2]. As the fluctuations of the energy and the interaction time are connected with uncertainty principle, $\Delta E \Delta \tau \geq \hbar$, the manipulation with quantum states of two oscillators becomes more difficult than in classical or semiclassical optics [3]. The quantum collapse and revival realization of two radiators is an attractive problem due to the possibilities of restoration of initially separated states of atom and electromagnetic field after interaction [1, 4]. With increasing degrees of freedom in one of the subsystems or with growing number of levels, the realization of reversible condition becomes more complicated from quantum statistical point of view.

There are a lot of works in the literature concerned with the restoration of initial states in the process of interaction. One of them is the quantum trapping effect described in paper [5], in which the reversibility of a three-level system is realized as a consequence of the stationary solution of Schrödinger equation $H_I \Psi(t) = 0$ and optical reversible phenomenon [4, 6], where during the interaction time, two quantum subsystems pass through a collapse and restore their initial states.

In this paper we analyze the possibility of states restoration of two quantum oscillators in interaction: a three-level atom and one mode of cavity field, which are off resonance each other. The realization of reversibility between these quantum subsystems becomes more complicated due to the fact that in accordance with uncertainty principle the energy and time of these subsystems in interaction must be larger than Planck constant. Taking into account this assumption, the time separability between such subsystems is more realistic in the case when the energy fluctuations of the system have large values. Thus, we can find the discrete interaction times, $\tau_1, \tau_2, \dots, \tau_n$, so that the probabilities of the restorations of same initial proprieties of the cavity field and atomic inversion become possible [4].

From quantum mechanical points of view we put the problem of reversibility in such interaction. In order to obtain this condition, the recursion relation between the decomposition

coefficients of wave functions on the Fock states is established. The condition is found for which the atom and cavity field restore all diagonal elements of density operator. If the transit time through the micro-cavity coincides with the reversible time, the atomic and cavity field subsystems restore all diagonal moments of reduced density matrix. Here, we demonstrate that in the non-resonant case the coupled phases between two subsystems remain unchanged. This can be observed after studying the nondiagonal elements of density matrix.

2. Temporal evolution of wave function for two photon excitation in the presence of detuning

Let us consider a three-level atom which enters in exact two photon resonance with single cavity mode relative to the ground $|g\rangle$ and excited $|e\rangle$ states as it is shown in Fig. 1.

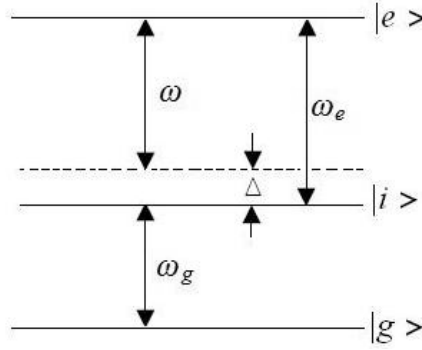


Fig. 1. Configuration of the energy levels of a three-level atom.

Assuming the energy is measured from the intermediate level of the atom, we study the influence of the detuning from resonance of its state. The general Hamiltonian of the system can be written

$$H = H_0 + H_i, \quad (1)$$

where

$$H_0 = \hbar\omega a^\dagger a + \hbar\omega_e |e\rangle\langle e| - \hbar\omega_g |g\rangle\langle g|, \quad (1a)$$

$$H_i = \hbar\lambda_1 (|i\rangle\langle g| a + a^\dagger |g\rangle\langle i|) + \hbar\lambda_2 (|e\rangle\langle i| a + a^\dagger |i\rangle\langle e|) \quad (1b)$$

are the free and interaction parts of the Hamiltonian; $E_g = -\hbar\omega_g$, $E_e = \hbar\omega_e$ represent the energetic positions of ground and excited levels of the atom. In Hamiltonian (1) the frequency ω corresponds to the resonance cavity mode described by annihilation a , and creation $a^\dagger$ field operators. The coefficients $\hbar\lambda_1 = (\vec{d}_{i,g}, \vec{g}(\omega))$ and $\hbar\lambda_2 = (\vec{d}_{i,e}, \vec{g}(\omega))$ represent the coupling energies between the cavity mode, level excitations $|i\rangle\langle g|$, $|e\rangle\langle i|$, and relaxation $|g\rangle\langle i|$, $|i\rangle\langle e|$ transitions. $\vec{d}_{i,g}$ and $\vec{d}_{i,e}$ are the transitions of dipole matrix elements; $\vec{g}(\omega)$ is the strength constant.

Taking into account that the frequency of cavity mode is off from resonance with $\Delta = \omega_e - \omega$ and $\Delta = \omega - \omega_g$, we can describe the behavior of atom-field system with the following model Hamiltonian

$$\tilde{H}_i = \hbar\Delta (|e\rangle\langle e| + |g\rangle\langle g|) + \hbar\lambda_1 (|i\rangle\langle g| a + a^\dagger |g\rangle\langle i|) + \hbar\lambda_2 (|e\rangle\langle i| a + a^\dagger |i\rangle\langle e|). \quad (2)$$

Let us prepare the atom at an earlier moment in the superposition of all three-level states in classical Ramsay zone [7]. Considering that at the time, $\tau = 0$, the atom enters in the cavity and the atomic and cavity field states are totally factorized

$$|\Psi(0)\rangle = \sum_{n=N_d}^{N_u} (\alpha|e\rangle + \beta|i\rangle + \gamma|g\rangle) \otimes S_n|n\rangle, \quad (3)$$

one can introduce the interaction with the cavity field in accordance with Hamiltonian (1). In the initial preparation of wave function (3) the following parameters are introduced: N_d and N_u represent the bottom and upper limits of the wave function decomposition on the Fock states and $|\alpha|^2, |\beta|^2, |\gamma|^2$ are the populations of the atomic levels. Taking into account the commutative propriety between the free and interaction part of the Hamiltonian (1), $[H_0, \tilde{H}_I] = 0$, the Schrödinger equation in the interaction picture

$$i\hbar\partial_\tau|\Psi(\tau)\rangle = \tilde{H}_I|\Psi(\tau)\rangle \quad (4)$$

for the coupled system “atom+cavity”, in accordance with the method proposed in [8] can be exactly solved

$$\begin{aligned} |\Psi(\tau)\rangle = & \sum_{n=N_d}^{N_u} \{ \{ a \frac{\lambda_2^2(n+1)}{\Theta_{n+1}} [e^{-i\Delta\tau/2} \cos(\Omega_{n+1} \frac{\tau}{2}) - i \frac{\Delta}{\Omega_{n+1}} e^{-i\Delta\tau/2} \sin(\Omega_{n+1} \frac{\tau}{2}) - e^{-i\Delta\tau}] S_n + a e^{-i\Delta\tau} S_n \\ & - 2\beta i \lambda_2 \frac{\sqrt{n+1}}{\Omega_{n+1}} e^{-i\Delta\tau/2} \sin(\Omega_{n+1} \frac{\tau}{2}) S_{n+1} \\ & + \gamma \frac{\lambda_1 \lambda_2 \sqrt{(n+1)(n+2)}}{\Theta_{n+1}} [e^{-i\Delta\tau/2} \cos(\Omega_{n+1} \frac{\tau}{2}) - i \frac{\Delta}{\Omega_{n+1}} e^{-i\Delta\tau/2} \sin(\Omega_{n+1} \frac{\tau}{2}) - e^{-i\Delta\tau}] S_{n+2} \} |e\rangle \\ & + \{ \beta e^{-i\Delta\tau/2} [\cos(\Omega_n \frac{\tau}{2}) + i \frac{\Delta}{\Omega_n} \sin(\Omega_n \frac{\tau}{2})] S_n - 2i\alpha \frac{\lambda_2 \sqrt{n}}{\Omega_n} e^{-i\Delta\tau/2} \sin(\Omega_n \frac{\tau}{2}) S_{n-1} \\ & - 2i\gamma \frac{\lambda_1 \sqrt{n+1}}{\Omega_n} e^{-i\Delta\tau/2} \sin(\Omega_n \frac{\tau}{2}) S_{n+1} \} |i\rangle \\ & + \{ \alpha \frac{\lambda_1 \lambda_2 \sqrt{n(n-1)}}{\Theta_{n-1}} [e^{-i\Delta\tau/2} \cos(\Omega_{n-1} \frac{\tau}{2}) - i \frac{\Delta}{\Omega_{n-1}} e^{-i\Delta\tau/2} \sin(\Omega_{n-1} \frac{\tau}{2}) - e^{-i\Delta\tau}] S_{n-2} \\ & - 2\beta i \lambda_1 \frac{\sqrt{n}}{\Omega_{n-1}} e^{-i\Delta\tau/2} \sin(\Omega_{n-1} \frac{\tau}{2}) S_{n-1} \\ & + \gamma \{ \frac{\lambda_1^2 n}{\Theta_{n-1}} [e^{-i\Delta\tau/2} \cos(\Omega_{n-1} \frac{\tau}{2}) - i \frac{\Delta}{\Omega_{n-1}} e^{-i\Delta\tau/2} \sin(\Omega_{n-1} \frac{\tau}{2}) - e^{-i\Delta\tau}] + e^{-i\Delta\tau} \} S_n \} |g\rangle \} |n\rangle, \quad (5) \end{aligned}$$

where $\Omega_n = \sqrt{\Delta^2 + 4(\lambda_1^2(n+1) + \lambda_2^2 n)}$ is the Rabi frequency and $\Theta_n = (\Omega_n^2 - \Delta^2)/4$.

During the interaction process the atom and cavity field are mixed in accordance with solution (5). We are interested to describe the class of field states for which the atomic population and photon distributions on the Fock states after the transit time, τ , remain unchanged as in initial states (3). In other words, the population probabilities of the field in the Fock states $|n\rangle$, $\langle\Psi(\tau)|n\rangle\langle n|\Psi(\tau)\rangle = |S_n|^2$, atom in the ground, $\langle\Psi(\tau)|g\rangle\langle g|\Psi(\tau)\rangle = |\gamma|^2$, interme-

diated, $\langle \Psi(\tau) | i \rangle \langle i | \Psi(\tau) \rangle = |\beta|^2$, and excited $\langle \Psi(\tau) | e \rangle \langle e | \Psi(\tau) \rangle = |\alpha|^2$ states are reversible. This field must have the special coefficients S_n , for which the reversibility is possible. Indeed, in accordance with wave function (5) this requirement can be satisfied for the evolution of the ground state amplitude during the transit time, τ . Thus, after interaction the amplitude on the ground state γ remains as in initial state (3), but with other phase: $\gamma'(\tau)S'_n(\tau) \rightarrow \gamma S_n e^{-i\Delta\tau}$:

$$\begin{aligned} & \left\{ \alpha \frac{\lambda_1 \lambda_2 \sqrt{n(n-1)}}{\Theta_{n-1}} [e^{-i\Delta\tau/2} \cos(\Omega_{n-1} \frac{\tau}{2}) - i \frac{\Delta}{\Omega_{n-1}} e^{-i\Delta\tau/2} \sin(\Omega_{n-1} \frac{\tau}{2}) - e^{-i\Delta\tau}] S_{n-2} \right. \\ & + \gamma \frac{\lambda_1^2 n}{\Theta_{n-1}} [e^{-i\Delta\tau/2} \cos(\Omega_{n-1} \frac{\tau}{2}) - i \frac{\Delta}{\Omega_{n-1}} e^{-i\Delta\tau/2} \sin(\Omega_{n-1} \frac{\tau}{2}) - e^{-i\Delta\tau}] S_n + e^{-i\Delta\tau} S_n \\ & \left. - 2\beta i \lambda_1 \frac{\sqrt{n}}{\Omega_{n-1}} e^{-i\Delta\tau/2} \sin(\Omega_{n-1} \frac{\tau}{2}) S_{n-1} \right\} |g\rangle |n\rangle = \gamma e^{-i\Delta\tau} S_n |g\rangle |n\rangle. \end{aligned}$$

Simultaneously, the evolution of the amplitude of the excited state must pass in the same initial state amplitude, $\alpha'(\tau)S'_n(\tau) = \alpha S_n e^{-i\Delta\tau}$, or

$$\begin{aligned} & \left\{ \alpha \frac{\lambda_2^2 (n+1)}{\Theta_{n+1}} [e^{-i\Delta\tau/2} \cos(\Omega_{n+1} \frac{\tau}{2}) - i \frac{\Delta}{\Omega_{n+1}} e^{-i\Delta\tau/2} \sin(\Omega_{n+1} \frac{\tau}{2}) - e^{-i\Delta\tau}] S_n + \alpha e^{-i\Delta\tau} S_n \right. \\ & + \gamma \frac{\lambda_1 \lambda_2 \sqrt{(n+1)(n+2)}}{\Theta_{n+1}} [e^{-i\Delta\tau/2} \cos(\Omega_{n+1} \frac{\tau}{2}) - i \frac{\Delta}{\Omega_{n+1}} e^{-i\Delta\tau/2} \sin(\Omega_{n+1} \frac{\tau}{2}) - e^{-i\Delta\tau}] S_{n+2} \\ & \left. - 2\beta i \lambda_2 \frac{\sqrt{n+1}}{\Omega_{n+1}} e^{-i\Delta\tau/2} \sin(\Omega_{n+1} \frac{\tau}{2}) S_{n+1} \right\} |e\rangle |n\rangle = \alpha e^{-i\Delta\tau} S_n |e\rangle |n\rangle. \end{aligned}$$

As follows, for both cases, we obtain the same recursion relation between the coefficients S_n

$$S_n = \frac{1}{2\beta\Theta_n} \left[i\Omega_n \frac{e^{-i\Delta\tau/2} - \cos(\Omega_n \frac{\tau}{2})}{\sin(\Omega_n \frac{\tau}{2})} - \Delta \right] [\alpha \lambda_2 \sqrt{n} S_{n-1} + \gamma \lambda_1 \sqrt{n+1} S_{n+1}] \quad (6)$$

Recursion relation (6) must be satisfied for all three-states of the atom at the same time. Introducing the coefficient, S_n , from this relation in the amplitude of the intermediate atomic state, $|i\rangle$, from solution (5) we obtain that the amplitude of the intermediate state also pass in the initial amplitude, but with a phase which depends on the field state

$$\beta'(\tau)S'_n(\tau)|i\rangle|n\rangle = -\beta e^{-i\Delta\tau} \frac{Z(n)}{Z^*(n)} S_n |i\rangle|n\rangle = -\beta e^{-i\Delta\tau + 2i \arg Z(n)} S_n |i\rangle|n\rangle, \quad (7)$$

where $Z(n) = \cos(\Omega_n \tau / 2) + i \frac{\Delta}{\Omega_n} \sin(\Omega_n \tau / 2) - e^{i\Delta\tau/2}$.

As a conclusion, we observe that recursion relation (6) will be satisfied for all atomic states and in the time moment, τ , the wave function of the system evaluates in a quasi-factorized state with the same probability amplitude as in the initial state

$$\Psi(\tau) = e^{-i\Delta\tau} \sum_{n=N_d}^{N_u} S_n [\alpha |e\rangle + \gamma |g\rangle - \beta \frac{Z(n)}{Z^*(n)} |i\rangle] |n\rangle. \quad (8)$$

In general case, the reversibility between two quantum oscillators can be analyzed in two cases:

a. Full separation, that corresponds to the moment for which the wave function of two interacting quantum subsystems (atom and EM field) becomes factor-

ized $\Psi(\tau) = \Psi_A(\tau) \otimes \Psi_{EM}(\tau)$. This assumption corresponds to zero value of the detuning, $\Delta = 0$, in which the full separability between these subsystems is realized. According to expression (7), the coefficient $Z(n)/Z^*(n) = 1$ and the wave function becomes totally factorized

$$\Psi(\tau) = \sum_{n=N_d}^{N_u} S_n |n\rangle \otimes [\alpha|e\rangle + \gamma|g\rangle - \beta|i\rangle].$$

b. Partial separation, in which two distinctive quantum subsystems can be regarded as one with wave function (3) decomposed on the atomic and field Fock states

$\Psi(0) = \sum_{n=N_d}^{N_u} \sum_{k=i,e,g} a_k S_n |n\rangle |k\rangle$, where the coefficient a_k coincides with α , β , and γ , respectively. In this preparation, the coefficient $|a_k|^2 |S_n|^2$, represents the probability to find the atom and field in the Fock state $|k\rangle |n\rangle$. If in the process of interaction this probability is not changed, the wave function can be expressed through the same amplitudes $a_k S_n$ as $\Psi(\tau) = \sum_{n=N_d}^{N_u} \sum_{k=i,e,g} a_k e^{i\phi(n)} S_n |n\rangle |k\rangle$. Using this definition, we observe that the phase factor $\phi(n)$

does not change the probability to find the atom and field in the Fock states $|k\rangle$ and $|n\rangle$: $|a_k S_n e^{i\phi(n)}|^2 = |a_k|^2 |S_n|^2$.

As follows from both definitions, the amplitude phases do not affect the diagonal matrix elements of the system after the interaction process. According to definition *b*, it is observed that the transition probabilities between $|k\rangle |n\rangle$ and $|k'\rangle |n'\rangle$ states are not factorized. From physical point of view, this effect can be clear, introducing the reduced density operator for atom and cavity field subsystems. Indeed, using the projector operator, $|\Psi(\tau)\rangle\langle\Psi(\tau)|$, one can obtain the reduced density matrix for atomic states, $W_A = Tr_{ph} \{ |\Psi(\tau)\rangle\langle\Psi(\tau)| \}$ or

$$W_A = [\alpha|e\rangle + \gamma|g\rangle][\alpha^*\langle e| + \gamma^*\langle g|] + |\beta|^2 |i\rangle\langle i| - \beta^* (\alpha|e\rangle + \gamma|g\rangle) \langle i| \sum_{n=N_d}^{N_u} |S_n|^2 \frac{Z^*(n)}{Z(n)} - \beta [\alpha^*\langle e| + \gamma^*\langle g|] |i\rangle \sum_{n=N_d}^{N_u} |S_n|^2 \frac{Z(n)}{Z^*(n)} \quad (9)$$

In analogical way, we can write the reduced density operator for the cavity field

$$W_{EM} = Tr_A \{ |\Psi(\tau)\rangle\langle\Psi(\tau)| \} = \sum_{n=N_d}^{N_u} \sum_{m=M_d}^{M_u} \left(1 + |\beta|^2 \left(\frac{Z_m^* Z_n}{Z_m Z_n^*} - 1 \right) \right) S_n S_m^* |n\rangle\langle m| \quad (10)$$

As follows from definitions (9) and (10) after the transit time, τ , the diagonal matrix elements coincide with their initial values, but the mixing phases between the atom and field of the nondiagonal matrix elements remain coupled. The last influences the cross fluctuations between the atomic and field subsystem.

3. Numerical results and discussions

The reversible condition described in this paper is more general than in paper [5], because it represents the evolution of the system in every moment. More than this, in the case when the detuning is larger $\Delta \rightarrow \infty$, $\lambda_1 = \lambda_2$ and the intermediate level is not populated, $\beta = 0$, the exact solution of wave function (5) and recursion relation (6) are reduced to the problem discussed in [9]

$$\begin{aligned}
 |\Psi(\tau)\rangle = & \sum_{n=N_d}^{N_u} \left\{ \left\{ \alpha \left[1 - 2i \frac{n+1}{2n+3} e^{-i\lambda\tau\sqrt{2n+3}} \sin(\lambda\tau\sqrt{2n+3}) \right] S_n \right. \right. \\
 & - 2i\gamma \frac{\sqrt{(n+1)(n+2)}}{2n+3} e^{-i\lambda\tau\sqrt{2n+3}} \sin(\lambda\tau\sqrt{2n+3}) S_{n+2} \Big\} |e\rangle \\
 & + \left\{ \gamma \left[1 - 2i \frac{n}{2n-1} e^{-i\lambda\tau\sqrt{2n-1}} \sin(\lambda\tau\sqrt{2n-1}) \right] S_n \right. \\
 & - 2i\alpha \frac{\sqrt{n(n-1)}}{2n-1} e^{-i\lambda\tau\sqrt{2n-1}} \sin(\lambda\tau\sqrt{2n-1}) S_{n-2} \Big\} |g\rangle \Big\} |n\rangle, \\
 S_n = & -\frac{\alpha}{\gamma} \sqrt{\frac{n-1}{n}} S_{n-2}.
 \end{aligned} \tag{11}$$

Let us find the population restoration of the excited level during the transit time interval $0 \leq \tilde{\tau} \leq \tau$. Taking into consideration wave function (5) it is observed that the mean value of populated number of the excited state, $|e\rangle\langle e|$, can be written in the following form

$$|\langle \Psi(\tilde{\tau}) | e \rangle|^2 = \sum_{n=N_d}^{N_u} |A_n(\tilde{\tau})|^2, \tag{12}$$

where

$$\begin{aligned}
 A_n(\tilde{\tau}) = & \alpha \frac{\lambda_2^2(n+1)}{\Theta_{n+1}} \left[e^{-i\Delta\tilde{\tau}/2} \cos\left(\Omega_{n+1} \frac{\tilde{\tau}}{2}\right) - i \frac{\Delta}{\Omega_{n+1}} e^{-i\Delta\tilde{\tau}/2} \sin\left(\Omega_{n+1} \frac{\tilde{\tau}}{2}\right) - e^{-i\Delta\tilde{\tau}} \right] \\
 & + e^{-i\Delta\tilde{\tau}} S_n - 2\beta i \lambda_2 \frac{\sqrt{n+1}}{\Omega_{n+1}} e^{-i\Delta\tilde{\tau}/2} \sin\left(\Omega_{n+1} \frac{\tilde{\tau}}{2}\right) S_{n+1}
 \end{aligned}$$

represents the transition amplitude on the excited state.

Figure 2 shows the atomic population probability on the excited state as functions of the atom transit time through the cavity.

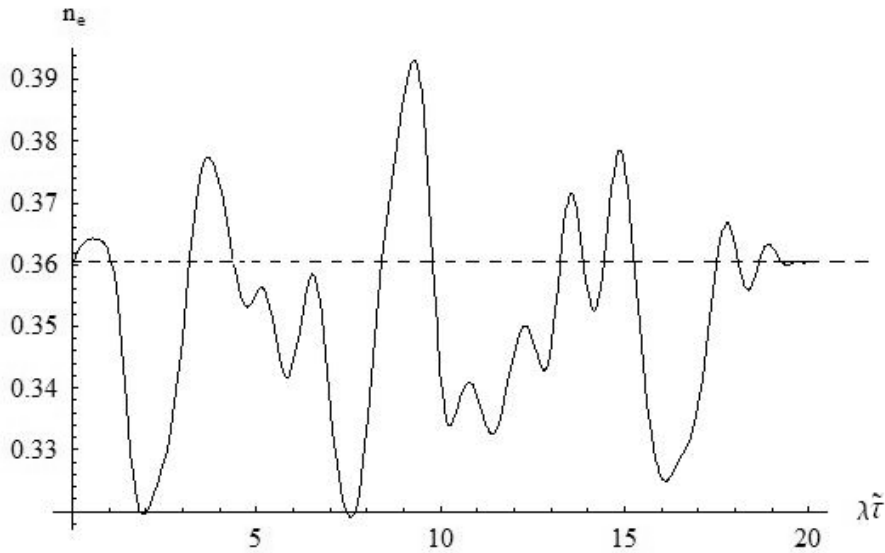


Fig. 2. Time evolution of probability of population on the excited state for the following values: $n=10$, $\Delta=1$, $\alpha=0.6$, and $\beta=0.2$.

Considering the situation for which, $\beta = 0$, we observe, that recursion relation (6) does not depend on the transit time $\tilde{\tau}$

$$\alpha\lambda_2\sqrt{n}S_{n-1} + \gamma\lambda_1\sqrt{n+1}S_{n+1} = 0,$$

and coincides with the trapping condition proposed in paper [5]. This case corresponds to the situation $H_I |\Psi(\tilde{\tau})\rangle = 0$ and, as follows from the numerical results, the probability of population on the excited (or ground) state remains unchanged during the interaction time $\tilde{\tau}$ (see Fig. 2, dashed line). In our problem, the atom is prepared in an aleatory superposition of states and in order to restore the initial values for $\beta \neq 0$, the system passes through the quantum oscillations. Here, we observe that the atom enters the cavity and leaves it at the time $\tilde{\tau} = 20/\lambda$ in the same population state ($n_e|_{\tilde{\tau}=0} = n_e|_{\tilde{\tau}=20} = 0.36$).

The influence of the detuning in the restoration process can be examined, studying the diagonal moments of the photon numbers

$$\langle n^p \rangle = \langle (a^\dagger a)^p \rangle = \sum_{n=N_d}^{N_u} n^p [A_n(\tilde{\tau})^2 + |B_n(\tilde{\tau})|^2 + |C_n(\tilde{\tau})|^2] \quad (13)$$

and non-diagonal moments of creation and annihilation operators

$$\langle a^p \rangle + \langle a^{+p} \rangle = \sum_{n=N_d}^{N_u} \sqrt{\frac{n!}{(n-p)!}} [A_n^*(\tilde{\tau})A_{n+p}(\tilde{\tau}) + B_n^*(\tilde{\tau})B_{n+p}(\tilde{\tau}) + C_n^*(\tilde{\tau})C_{n+p}(\tilde{\tau})] + hc, \quad (14)$$

where $p = 1, 2, \dots$ is an integer number. According to exact solution (5) the amplitudes $B_n(\tilde{\tau})$, and $C_n(\tilde{\tau})$ are defined as

$$\begin{aligned} B_n(\tilde{\tau}) = & -\alpha \frac{2i\lambda_2\sqrt{n}}{\Omega_n} e^{-i\Delta\tilde{\tau}/2} \sin\left(\Omega_n \frac{\tilde{\tau}}{2}\right) S_{n-1} + \beta e^{-i\Delta\tilde{\tau}/2} \left[\cos\left(\Omega_n \frac{\tilde{\tau}}{2}\right) + i \frac{\Delta}{\Omega_n} \sin\left(\Omega_n \frac{\tilde{\tau}}{2}\right) \right] S_n \\ & - \gamma \frac{2i\lambda_1\sqrt{n+1}}{\Omega_n} e^{-i\Delta\tilde{\tau}/2} \sin\left(\Omega_n \frac{\tilde{\tau}}{2}\right) S_{n+1}, \\ C_n(\tilde{\tau}) = & \alpha \frac{\lambda_1\lambda_2\sqrt{n(n-1)}}{\Theta_{n-1}} \left[e^{-i\Delta\tilde{\tau}/2} \cos\left(\Omega_{n-1} \frac{\tilde{\tau}}{2}\right) - i \frac{\Delta}{\Omega_{n-1}} e^{-i\Delta\tilde{\tau}/2} \sin\left(\Omega_{n-1} \frac{\tilde{\tau}}{2}\right) - e^{-i\Delta\tilde{\tau}} \right] S_{n-2} \\ & - 2\beta i\lambda_1 \frac{\sqrt{n}}{\Omega_{n-1}} e^{-i\Delta\tilde{\tau}/2} \sin\left(\Omega_{n-1} \frac{\tilde{\tau}}{2}\right) S_{n-1} \\ & + \gamma \left\{ \frac{\lambda_1^2 n}{\Theta_{n-1}} \left[e^{-i\Delta\tilde{\tau}/2} \cos\left(\Omega_{n-1} \frac{\tilde{\tau}}{2}\right) - i \frac{\Delta e^{-i\Delta\tilde{\tau}/2}}{\Omega_{n-1}} \sin\left(\Omega_{n-1} \frac{\tilde{\tau}}{2}\right) - e^{-i\Delta\tilde{\tau}} \right] + e^{-i\Delta\tilde{\tau}} \right\} S_n. \end{aligned}$$

As follows from the last section all diagonal moments are restored in the presence of detuning, while the non-diagonal moments remain non-restored. Only in the case of good resonance the full restoration of all matrix elements is possible.

The time evolution of the mean photon numbers fluctuations, σ , is defined by

$$\sigma = \frac{\langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2}{\langle \hat{n} \rangle}, \quad (15)$$

and is plotted in Fig. 3.

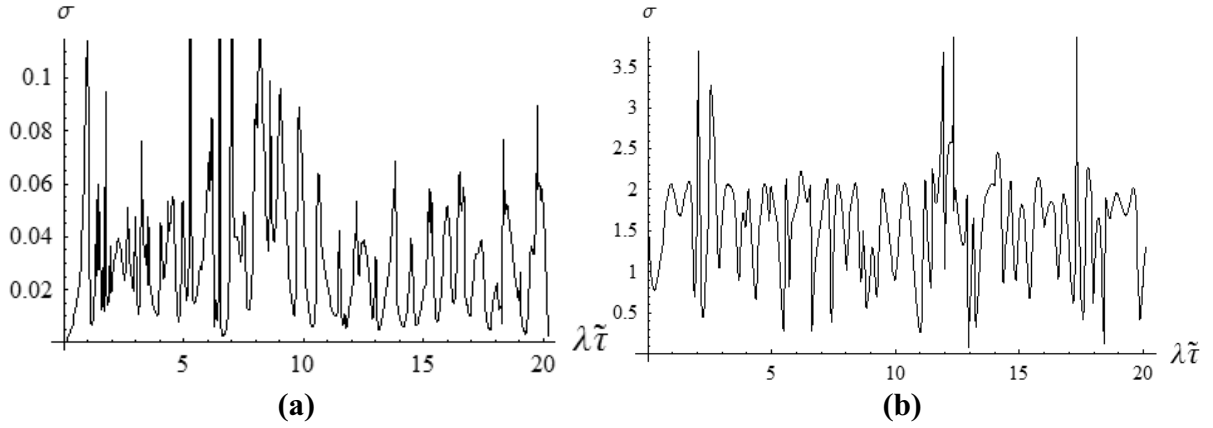


Fig.3. Time dependence of relative fluctuations σ for: (a) $n=10$, $\Delta=0.1$, $\alpha=0.9$, $\beta=0.4$; (b) $n=10$, $\Delta=10$, $\alpha=0.8$, and $\beta=0.3$.

As a consequence, the cavity field proprieties depend on recursion relation (6) and remain in the initial state after the transit time. Therefore, for small values of detuning the photon statistics is sub-Poissonian as in Fig. 3a. On the contrary, with increasing parameter Δ the quantum electromagnetic cavity proprieties obey super-Poissonian statistics (see Fig. 3b).

Now, let us analyze the normalized fluctuations of the field quadratures, $\hat{A}_x = (a^+ + a)/2$ and $\hat{A}_y = i(a^+ - a)/2$ [10], which satisfy the following commutation relation $[\hat{A}_x, \hat{A}_y] = i/2$. Taking into account the uncertainty principle $\Delta_x \Delta_y \geq 1/2$, one can introduce the square values of quadratures $\delta_i^2 = (\langle : A_i^2 : \rangle - \langle A_i \rangle^2) / \langle A_i \rangle^2$, where i represents the real x or imaginary y part of the field quadrature and $: A_i^2 :$ the normal produce of the operators a^+ and a . Using these definitions we obtain the following expressions for the fluctuation in quadratures

$$\begin{aligned} \delta_x^2 &= \frac{\langle a^2 \rangle + \langle a^{+2} \rangle + 2\langle a^+ a \rangle}{(\langle a^+ \rangle + \langle a \rangle)^2} - 1, \\ \delta_y^2 &= \frac{\langle a^2 \rangle + \langle a^{+2} \rangle - 2\langle a^+ a \rangle}{(\langle a^+ \rangle - \langle a \rangle)^2} - 1. \end{aligned} \quad (16)$$

Figure 4 illustrates the time dependence of expressions (16).

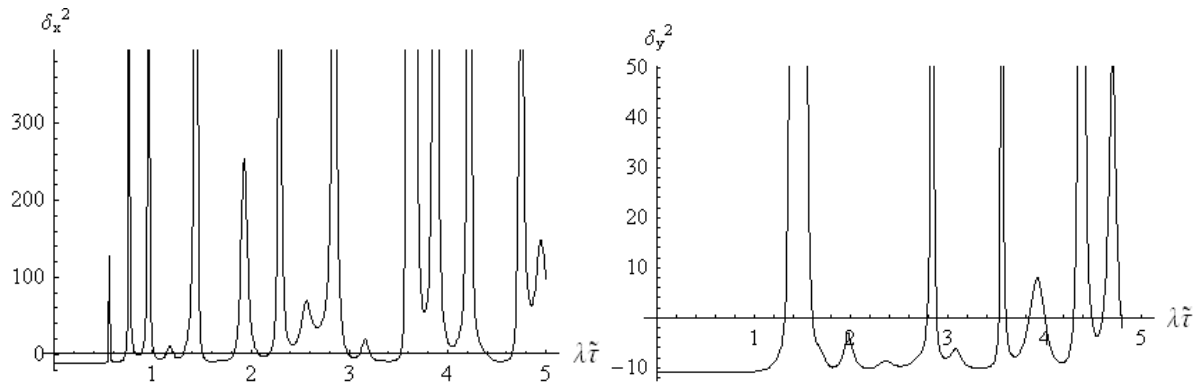


Fig. 4. Fluctuations in quadratures δ_x^2 and δ_y^2 as function of $\lambda\tau$ for the following parameters: $n=10$, $\Delta=1$, $\alpha=0.8$, and $\beta=0.3$.

For non-zero detuning case, $\Delta \neq 0$, the fluctuations of quadratures in the initial and final moments differ. In other words, the atomic transit through the prepared field state according to reversible condition (6) changes the nondiagonal moments of field (14). In the opposite case, when the detuning from resonance takes zero value $\Delta = 0$, the full restoration of diagonal and nondiagonal moments is realized (see Fig. 5).

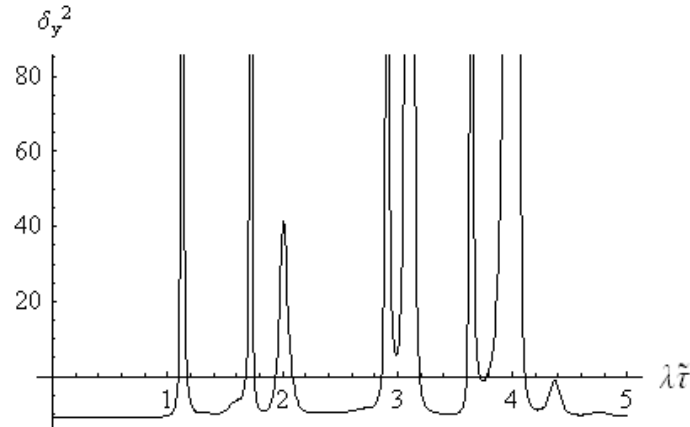


Fig. 5. The same as in Fig. 4 except $\Delta=0$.

4. Conclusions

In this paper we have found the reversible conditions for which two quantum subsystems in interaction can restore their initial states. It is demonstrated that the phases coupling between the cavity field and atomic inversion remain unchanged. As follows from reduced density operator, after the interaction process the atom and cavity subsystem can restore only their initial diagonal matrix elements (atomic population, mean number of photons, and its fluctuations), but the nondiagonal elements remain coupled. This effect was called partial restoration of two quantum interaction subsystems. We study the reversibility taking into account the actual experimental realizations [11, 12] with the real detuning between the field and intermediate state. For experimental realization of these effects in maser regime, the Rydberg atoms like Rb can be proposed, which are prepared in superposition states relative to the cascade transitions $40S_{1/2} \leftrightarrow 39P_{1/2} \leftrightarrow 39S_{1/2}$ [11]. The electric dipole matrix elements in this cascade configuration are exceedingly large. These transitions correspond to the millimeter wave length with cascade transition frequencies $\nu_{sp} = 68.41587 \text{ GHz}$ and $\nu_{sp} = 68.3768 \text{ GHz}$ for that it is necessary to realize the high-Q bimodal cavity efficiently coupled to the atomic dipoles. Off-resonance behavior and some proprieties of cavity field were analyzed.

Acknowledgements

This work was supported by National Grant No. 06.408.013F and Young Scientists Grant No. 07.408.31 of the Academy of Sciences of Moldova.

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