OSCILLATIONS OF SQUEEZING IN MICROCAVITY

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Abstract

We study the interaction between the single-mode electromagnetic field and the pair of indistinguishable two-level atoms via the intensity-dependent coupling. This problem is equivalent to the equidistant three-level atom with equal dipole moment matrix transition elements between the adjacent levels. The exact analytical solution for the atom-field state-vector is obtained assuming that at the initial moment the field is in the Holstein-Primakoff SU (1,1) coherent state. The quantum-statistical and squeezing properties of the field are investigated. The obtained results are compared with those for the single two-level atom. It is observed that the exact periodicity of the field squeezing that takes place in the case of the single two-level atom is violated in the case of the pair of cold atoms. In other words, the exact periodicity of the physical quantities can be destroyed only if the radiation field interacts with a system of more than one two-level atom.

1. Introduction

At present great attention is paid to the cooperative interaction of radiators with quantized electromagnetic field and special attention is paid to the two-atom systems interacting with radiation field. For example, Tanas and Ficek investigated two-atom entanglement induced by nonclassical two-photon correlations [1]. In this paper it was demonstrated that the pair of two-level atoms interacting with a squeezed vacuum field can exhibit stationary entanglement associated with nonclassical two-photon correlations characteristic of the squeezed vacuum field. Lately, in Ref. 2 Chimczak et al proposed a scheme to teleport an entangled state of two Lambda-type atoms via photons. It was shown how to manipulate a state of many Lambda-type atoms trapped in a cavity.

On the other hand, with recent rapid advances in ion-trapping technology two-slit experiments, requiring the isolation and localization of two atoms are now possible. Thus, a Young two-slit experiment with trapped and laser-cooled ions has been reported [3]. In this situation it is important to study the interaction between the single-mode field and the pair of cold atoms via intensity-dependent coupling [4-5]. We suppose that two-level atoms in the pair are indistinguishable. This problem generalizes the Jaynes-Cummings model (JCM) with the intensity-dependent coupling interacting with Holstein-Primakoff SU (1,1) coherent state (CS) [6]. It should be noted that the pair of indistinguishable two-level atoms is equivalent to the three-level equidistant model with equal dipole moment matrix transition elements between the adjacent levels [7]. In this situation the states of the atomic pair can be described in the three-level state representation in the following way: the ground state $|g\rangle$ describes the case when both atoms of the pair are in the ground state, the first excited state $|e_1\rangle$ is equivalent to the case in which the first two-level atom of the pair is in the ground state and the second one is in the excited state and in the second excited state $\left|e_2\right\rangle$ both atoms are in the excited state. We examine the special case of the Holstein-Primakoff realization of the SU (1,1) Lie algebra [8-9]. Much attention in our paper is paid to the squeezing properties of the radiation field. It should be noted that in Ref. 6 it was found that if the variance of the field quadrature is squeezed at the initial moment, then the squeezing can recur later. Moreover, the revivals of the radiation squeezing are strictly periodical for any value of the initial squeezing. On the other hand, in the two-photon JCM the revivals of squeezing are not periodical [10]. Gerry has found that the higher the initial squeezing, the more regular the oscillations become. In this case it is instructive to investigate the squeezing oscillations in the two-atom model and to compare them with the strictly periodical revivals of squeezing for the single-atom one [6].

2. The Hamiltonian of the atom-field system

Let us first represent the Hamiltonian describing the interaction of the single-mode field with the pair of cold two-level atoms (or equidistant three-level atom) via intensity-dependent coupling in the rotating-wave approximation

$$H = \hbar\omega_0 S_z + \hbar\omega a^+ a + \hbar\lambda(R^+ S^- + R S^+), \tag{1}$$

where

$$\begin{aligned} S_z &= \left|e_2\right\rangle\left\langle e_2\right| - \left|g\right\rangle\left\langle g\right|, \\ S^+ &= \sqrt{2}\left(\left|e_2\right\rangle\left\langle e_1\right| + \left|e_1\right\rangle\left\langle g\right|\right), \\ S^- &= \sqrt{2}\left(\left|e_1\right\rangle\left\langle e_2\right| + \left|g\right\rangle\left\langle e_1\right|\right) \end{aligned} \tag{2}$$

are the atomic operators, satisfying the commutation relations for the SU (2) algebra [11-12]

$$[S_z, S^\pm] = \pm S^\pm, \quad [S^+, S^-] = 2S_z, \tag{3}$$

ω_0 and ω are the frequencies of the atomic transitions and the radiation field, respectively, λ is the atom-field coupling constant, $R = a\sqrt{N}$, $R^+ = \sqrt{N}a^+$, where $N = a^+ a$ is the photon number operator. To represent this Hamiltonian through the SU (1,1) symmetry group generators we introduce the following operators

$$K^+ = R^+, \; K^- = R, \; K_0 = N + 1/2, \tag{4}$$

which obey the commutation relations for the SU(1,1) Lie algebra [6, 11-12]

$$[K_0, K^\pm] = \pm K^\pm, \quad [K^+, K^-] = -2K_0. \tag{5}$$

In this situation the Hamiltonian (1) may be rewritten in terms of the SU (1,1) Lie algebra operators in the following manner

$$H = \hbar\omega_0 S_z + \hbar\omega\left(K_0 - \frac{1}{2}\right) + \hbar\lambda(K^+ S^- + K^- S^+). \tag{6}$$

It should be emphasized that this Hamiltonian has the same form as that considered in Ref. 10. But in our model the realization of the operators K_0 and $K^\pm$ to a marked degree differs from the two-photon JCM studied by Gerry

$$K_0 = \frac{N + 1/2}{2}, \quad K^+ = \frac{a^{+2}}{2}, \quad K^- = \frac{a^2}{2}. \tag{7}$$

3. The quantum-statistical and squeezing properties of the radiation field

Let us suppose that at the initial moment $t=0$ the atomic system is in the first excited state $|e_1\rangle$ and the single-mode field is in the Holstein-Primakoff SU (1,1) coherent state $|\xi\rangle$ [6, 13-14]

$$|\xi\rangle=(1-|\xi|^2)^k\sum_{m=0}^{\infty}\left\{\frac{\Gamma(m+2k)}{m!\Gamma(2k)}\right\}^{1/2}\xi^m|m,k\rangle\,, \tag{8}$$

where $|m,k\rangle$ are the eigenvectors of the Casimir operator $C=K_0^2-\frac{K_+K_-+K_-K_+}{2}=$ $=k(k-1)I$, where k is the so-called Bargmann index and $\xi=|\xi|\exp(i\varphi)$, $(0\le|\xi|\le 1)$. As in the present paper $k=1/2$ the SU (1,1) CS can be written through the usual oscillator number states [6]

$$|\xi\rangle=(1-|\xi|^2)^{1/2}\sum_{n=0}^{\infty}\xi^n|n\rangle=\sum_{n=0}^{\infty}Q_n|n\rangle. \tag{9}$$

This CS is the special (one-photon) case of the multiboson Holstein-Primakoff SU (1,1) CS [15-17]. In this situation the initial state-vector $|\psi(t=0)\rangle$ of the coupled atom-field system can be written by the following

$$|\psi(t=0)\rangle=|\xi\rangle\otimes|e_1\rangle=\sum_{n=0}^{\infty}Q_n|e_{1,}n\rangle. \tag{10}$$

To study the quantum statistical properties of the radiation field it is necessary to know the time-evolution of the state-vector for the atom-field system. By using the Schrodinger equation

$$i\hbar\frac{d}{dt}|\psi(t)\rangle=H|\psi(t)\rangle \tag{11}$$

we obtain the exact analytical solution for $|\psi(t)\rangle$ in the resonance case ($\omega=\omega_0$)

$$\begin{aligned}|\psi(t)\rangle=&\sum_{n=0}^{\infty}e^{-i\omega_0\{S_z+n\}t}Q_n\{\cos(\sqrt{2n^2+2n+1}\tau)|e_1,n\rangle-\\&-\frac{i(n+1)}{\sqrt{2n^2+2n+1}}\sin(\sqrt{2n^2+2n+1}\tau)|g,n+1\rangle-\frac{in}{\sqrt{2n^2+2n+1}}\sin(\sqrt{2n^2+2n+1}\tau)|e_{2,}n-1\rangle,\end{aligned} \tag{12}$$

where $\tau=\sqrt{2}\lambda t$.

Let us now study the quantum-statistical properties of the single-mode cavity field interacting with pair of indistinguishable two-level atoms. From physical point of view it is important to compare these properties with those obtained for the single-atom model. Thus, for the mean photon number $\langle n\rangle$ and the atomic population inversion (API) $\langle S_z\rangle$ one can obtain the following expressions

$$\langle n\rangle=\frac{|\xi|^2}{1-|\xi|^2}+(1-|\xi|^2)\sum_{n=0}^{\infty}(|\xi|^2)^n\sin^2(\sqrt{2n^2+2n+1}\tau)\frac{2n+1}{2n^2+2n+1} \tag{13}$$

and

$$\langle S_z \rangle = -(1-|\xi|^2)\sum_{n=0}^{\infty}(|\xi|^2)^n \sin^2(\sqrt{2n^2+2n+1}\tau)\frac{2n+1}{2n^2+2n+1}. \tag{14}$$

From Eqs. (13) and (14) it is clearly seen that the exact periodicity of the mean photon number and the API is violated in the case of the equidistant three-level atom (or the pair of indistinguishable two-level atoms) in contrast with the two-level one, in which the API exhibits the exact periodicity of the population revivals [6]. It is also desirable to investigate the quantum fluctuations in the mean photon number σ

$$\sigma = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle}, \tag{15}$$

where $\langle n^2 \rangle$ is given by

$$\langle n^2 \rangle = \frac{|\xi|^2(1+|\xi|^2)}{(1-|\xi|^2)^2} + (1-|\xi|^2)\sum_{n=0}^{\infty}(|\xi|^2)^n \sin^2(\sqrt{2n^2+2n+1}\tau)\frac{6n^2+4n+1}{2n^2+2n+1}. \tag{16}$$

In this case both Sub- and Super-Poissonian photon statistics take place. Moreover, the fluctuations in the mean number of photons σ have the oscillatory behaviour. In this situation it should be noted that the similar oscillatory behaviour of the fluctuations was observed in Ref. 7, 18-20 in the studying of trapping effect for the three-level atom passing through the resonant cavity.

Let us investigate the squeezing properties of the radiation field [21-22] in the case of the two-atom system in order to compare our results with those for the single two-level atom [6]. In this situation it should be noted that in the coherent state the fluctuations of the real and imaginary field quadratures X_1 and X_2 are equal to 1/4, where

$$X_1 = \frac{ae^{i\omega t} + a^+ e^{-i\omega t}}{2}, \quad X_2 = \frac{ae^{i\omega t} - a^+ e^{-i\omega t}}{2i}. \tag{17}$$

It is well-known that the squeezed state is the state, in which the fluctuations of the field quadratures are smaller than those in the coherent state. To study the squeezing we introduce the functions $S_i(t)$, $(i=1,2)$

$$S_i(t) = \frac{\langle (\Delta X_i)^2 \rangle - \langle (\Delta X_i)^2 \rangle_{\mathrm{coh}}}{\langle (\Delta X_i)^2 \rangle_{coh}} = 4\langle (\Delta X_i)^2 \rangle - 1. \tag{18}$$

Here $\langle (\Delta X_i)^2 \rangle = \langle X_i^2 \rangle - \langle X_i \rangle^2$ and $\langle (\Delta X_i)^2 \rangle_{coh} = 1/4$. In this situation the squeezing condition is

$$-1 < S_i < 0.$$

To examine the squeezing properties numerically it is necessary to know the mean values of the single-mode field operators a, a^2, a^+ and a^{+2}. For these mean values we obtain the following result

$$\langle a \rangle = e^{-i(\omega t-\varphi)}A_1(t), \ \langle a^2 \rangle = e^{-2i(\omega t-\varphi)}A_2(t),$$
$$\langle a^+ \rangle = e^{i(\omega t-\varphi)}A_1(t), \ \langle a^{+2} \rangle = e^{2i(\omega t-\varphi)}A_2(t), \tag{19}$$

where

$$A_1(t) = |\xi| \sum_{n=0}^{\infty} P_n \Bigg[\sqrt{n+1} \cos(\sqrt{2n^2+2n+1}\tau) \cos(\sqrt{2(n+1)^2+2(n+1)+1}\tau) +$$

$$+ \left\{(n+2)^{3/2} + n^{3/2}\right\}(n+1) \frac{\sin(\sqrt{2n^2+2n+1}\tau)\sin(\sqrt{2(n+1)^2+2(n+1)+1}\tau)}{\sqrt{2n^2+2n+1}\sqrt{2(n+1)^2+2(n+1)+1}} \Bigg], \tag{20}$$

$$A_2(t) = |\xi|^2 \sum_{n=0}^{\infty} P_n \Bigg[\sqrt{(n+1)(n+2)} \cos(\sqrt{2n^2+2n+1}\tau) \cos(\sqrt{2(n+2)^2+2(n+2)+1}\tau) +$$

$$+ \left\{(n+3)^{3/2}\sqrt{(n+2)}(n+1) + n^{3/2}\sqrt{n+1}(n+2)\right\} \times$$

$$\times \frac{\sin(\sqrt{2n^2+2n+1}\tau)\sin(\sqrt{2(n+2)^2+2(n+2)+1}\tau)}{\sqrt{2n^2+2n+1}\sqrt{2(n+2)^2+2(n+2)+1}} \Bigg]. \tag{21}$$

Here $P_n = |Q_n|^2 = (1-|\xi|^2)|\xi|^{2n}$ With the help of these expressions for the functions $S_1(t)$ and $S_2(t)$ we obtain the following result

$$S_1(t) = 2[A_0(t) - A_2(t)] + 4\cos^2(\varphi)[A_2(t) - A_1^2(t)], \tag{22}$$

$$S_2(t) = 2[A_0(t) - A_2(t)] + 4\sin^2(\varphi)[A_2(t) - A_1^2(t)], \tag{23}$$

where $A_0(t) = \langle n \rangle$.

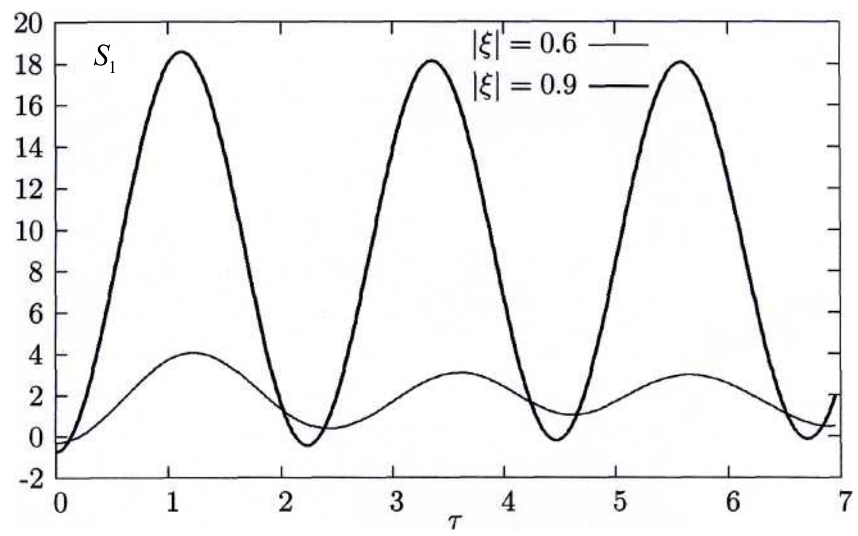

Figure 1. Time-evolution of the function $S_1(\tau)$ for $|\xi| = 0.6$ and 0.9.

In figure 1 we present S_1 as a function of the dimensionless time τ for different values of $|\xi| = 0.6$ and 0.9. From this figure it is clearly seen that the exact periodicity of the squeezing revivals is violated in the case of the two-atom system, whereas in the single-atom model the revivals of squeezing are strictly periodical for any value of initial squeezing [6].

In conclusion, we have shown that squeezed states of light can be generated in the interaction of quantized field of the cavity with pair of cold atoms via intensity-dependent coupling. It is found that with the increasing of the initial mean photon number

$\langle n(t=0)\rangle = |\xi|^2/(1-|\xi|^2)$ the squeezing increases with the maximum value about 75%. It is worthwhile to point out that the exact periodicity of the squeezing revivals is violated in the proposed model. In other words, the exact periodicity of squeezing oscillations is violated if the single-mode field interacts with a system of more than one two-level atom.

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