RELAXATION POLARIZATION DESCRIPTION BY OVERDAMPED LINEAR OSCILLATOR MODEL

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(Received 6 October 2006)

Authors show that the relaxation polarization theory can be developed on the basis of linear oscillator model. Spectral formulas for overdamped and strongly overdamped linear oscillator are obtained. The range of application of obtained formulas is determined. Main features of the relaxation polarization are found.

1. Introduction

In accordance with traditional point of view, the elastic and the relaxation (inelastic) polarizations have different physical nature [1-6]. The theory of the elastic polarization is based on the linear oscillator model. The frequency dependence of elastic polarization is described by the well-known spectral Lorentz formulas

$$\widetilde{A}(\omega) = \frac{A_0}{\omega^2 - \omega_0^2 + i2\beta\omega} = A_0\left(\frac{(\omega^2 - \omega_0^2)}{(\omega^2 - \omega_0^2)^2 + 4\beta^2\omega^2} - i\frac{2\beta\omega}{(\omega^2 - \omega_0^2)^2 + 4\beta^2\omega^2}\right), \quad (1)$$

where $\widetilde{A}(\omega)$ is the complex spectral function, A_0 is the amplitude, ω is the cyclic frequency of an external electric field change, ω_o is the natural frequency, β is the damping coefficient, i is the complex unite.

The relaxation polarization theory is based on the Skanavy statistical model [2]. The relaxation polarization frequency dependence is described by the spectral Debye formulas

$$\widetilde{\varepsilon}(\omega) = \varepsilon_\infty + (\varepsilon_s - \varepsilon_\infty)\left(\frac{1}{1+\omega^2\tau^2} - i\frac{\omega\tau}{1+\omega^2\tau^2}\right), \quad (2)$$

where $\widetilde{\varepsilon}(\omega)$ is the complex dielectrical permittivity, τ is the relaxation time, ε_∞ is high frequency limit of the permittivity, ε_s is its low frequency limit. Whereas the Lorentz formulas are the direct sequence of the linear oscillator model, the Debye formulas are not a sequence of the statistical model by Skanavy.

2. Theory

We suppose that the relaxation polarization can be described by the overdamped linear oscillator model. Let's consider the linear oscillator forced by harmonic internal force. The canonical form of equation for forced damped oscillator is [7]:

$$\frac{1}{\omega_0^2}\frac{d^2x}{dt^2} + \tau\frac{dx}{dt} + x = \frac{F(\omega,t)}{m\omega_0^2}, \quad (3)$$

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where x is the particles coordinate, t is the time, $\tau=2\beta/\omega_0^2$ is the time constant. Let's assign $F(\omega,t)=qE_0e^{i\omega t}$. It is the force acting on a charged particle with the charge q inside a dielectric by harmonic field with amplitude E_0. Solving of Eq. (3) gives $x(t,\omega)=\widetilde{A}(\omega)\cdot e^{i\omega t}$ [8, 9], where the complex amplitude is

$$\widetilde{A}(\omega)=\frac{qE_0}{m\omega_0^2}\frac{1}{-\omega^2/\omega_0^2+i\omega\tau+1}. \qquad (4)$$

This is another form of the spectral Lorentz formulas.

Let's consider two cases for forced damped oscillator:

Case 1. The oscillator is overdamped, $\beta/\omega_0 >> 1$. The inertia force becomes negligible in comparison with the damping [9]. Hence, we ignore the first term in left part of Eq. (3):

$$\tau\frac{dx}{dt}+x=\frac{qE_0}{m\omega_0^2}e^{i\omega t}. \qquad (5)$$

This is the motion equation for the weightless particle moving in the high damping medium. Solving Eq. (5) for the spectral function we obtain:

$$\widetilde{a}(\omega)=\frac{qE_0}{m\omega_0^2}\frac{1}{1+i\omega\tau}=\frac{qE_0}{m\omega_0^2}\left(\frac{1}{1+\omega^2\tau^2}-i\frac{\omega\tau}{1+\omega^2\tau^2}\right), \qquad (6)$$

where $\widetilde{a}(\omega)$ is the spectral function of overdamped oscillator. In order to determine the boundary condition for an application domain of spectral formula (6) let's consider the ratio of modules of (6) and (4) at frequency $\omega=\omega_0$

$$\frac{\sqrt{1+\omega^2\tau^2}}{\sqrt{(1-\omega^2/\omega_0^2)^2+\omega^2\tau^2}}=\frac{\sqrt{1+\omega_0^2\tau^2}}{\omega_0\tau}\le 1{,}01. \qquad (7)$$

Solving this inequality for τ we have:

$$\tau\ge\frac{7}{\omega_0} \qquad \text{or} \qquad \beta\ge 3.5\omega_0. \qquad (8)$$

Thus, under this condition spectral formulas (4) and (6) coincide to within 1%. Note, that we can use the equality $\beta>\omega_0$ as the approximate boundary to within 20%.

Case 2. The oscillator is strongly overdamped, $\beta/\omega_0^2 >> 1$. Both the inertia force and the quasielastic force become negligible in comparison with the friction force. Hence, we ignore the first and the third terms of Eq. (3) and find the spectral function for this case

$$\widetilde{a}(\omega)=\frac{q^2}{m\omega_0^2}\frac{1}{i\omega\tau}. \qquad (9)$$

One can see, this is a hyperbola. Function (9) is purely imaginary, i.e. vibration in the super high damping medium leads to an increase of energy loss only.

Thus, we have obtained spectral formula (6) for overdamped oscillator, that is similar to the Debye formulas. This fact allows us to draw a conclusion that the relaxation polarization can be described within the framework of linear oscillator model.

Let's make a note concerning terminology. In this study the linear oscillator model based on the elastic force concept is applied to describe the relaxation polarization. Hence, terms elastic and inelastic become unfounded. That's why, below we use the term resonant for elastic polarization and do not use the term inelastic for the relaxation polarization.

Let's find the relation of the linear oscillation model to the statistical theory. We make use of basic statements of the Skanavy statistical thermal polarization theory [2]. Let's assume, as he did, that a loosely bound charged particle locates in a one-dimensional double-

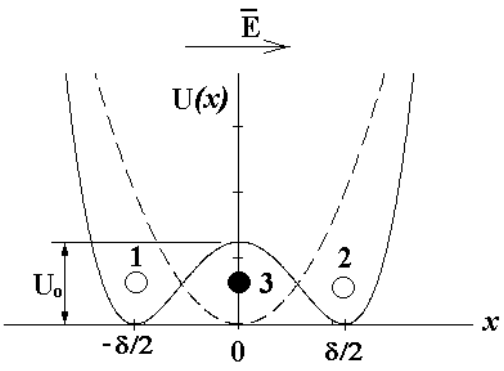

Fig. 1. ——— the double-well potential.
- - - - - parabolic-well potential with equilibrium position 3.

potential well (see Fig. 1). Equilibrium positions inside the potential well are separated by low potential barrier of height *U*. Minima are at $x_{1,2} = \pm\delta/2$, the barrier is at *x=0*. Due to the thermal motion the particle may jump from one equilibrium position to another. The concentration of loosely bound charged particles in the dielectric is equal to n_0, the concentration of particles at positions 1 and 2 are $n_1 = n_2 = n_o/2$. It is obvious that a dipole moment of the micro volume containing the potential well is not zero due to the asymmetrical charge location. However, without the external electric field the dipole moment of dielectric as a whole is zero. It means, that the polarization of dielectric is not determined by location of an isolated particle. It's determined by the location of n_0 particles. Thus, in order to describe the behavior of the particle ensemble we introduce the concept of an average particle. It has the average mass, the mass center coordinate and the charge. The application of an external constant electric field to the dielectric causes the increase of particle concentration in position 2 by value Δn and its decrease in position 1 by the same value. It is obvious that in this case the average mass center coordinate is

$$x(\Delta n) = \frac{\delta}{2}\frac{n_2 - n_1}{n_o} = \delta\frac{\Delta n}{n_o}. \qquad (10)$$

The maximum value of the average coordinate is $x(\Delta n) = \delta/2$. Hence, the Skanavy statistical model describes the motion of the average particle located in single-potential well (see Fig. 1). Thus, the average particle displacement will cause appearance of the dielectric dipole moment and consequently it will cause appearance of the returning force. Let's assume that the returning force is quasielastic, i.e. one-potential well is parabolic and Eq. (5) is the motion equation of the average charged particle. Main argument for this assumption is similarity of the spectral formula for overdamped oscillator (6) and the Debye formulas.

Let's find a relation of function (6) to the Debye formulas. In order to find a complex dielectric permittivity we use the Borhn formula [2]:

$$\tilde{\varepsilon}(\omega) = \varepsilon_\infty + \frac{\tilde{\alpha} n}{\varepsilon_0}, \qquad (11)$$

where ε_∞ is the high-frequency limit of the permittivity, $\tilde{\alpha}$ is the complex polarizability per a particle, *n* is the loosely bound particle concentration in a dielectric, ε_o is the permittivity of empty space. On the one hand the polarizability is a proportionality coefficient between elementary dipole moment and an electrical field strength $p = \tilde{\alpha}(\omega)E(t,\omega)$, on the other hand $p=x(t,\omega)\cdot q$ [3]. Therefore, using function (6) we obtain

$$\tilde{\alpha}(\omega) = \frac{q}{m\omega_0^2}\frac{1}{1+i\omega\tau}. \qquad (12)$$

Hence,

$$\tilde{\varepsilon}(\omega) = \varepsilon_\infty + \frac{q^2 n}{\varepsilon_0 m\omega_0^2}\frac{1}{1+i\omega\tau} = \varepsilon_\infty + \frac{q^2 n}{\varepsilon_0 m\omega_0^2}\left(\frac{1}{1+\omega^2\tau^2} - i\frac{\omega\tau}{1+\omega^2\tau^2}\right). \qquad (13)$$

Thus, we can introduce a notation relating spectral formula (6) to the Debye formulas

$$(\varepsilon_s - \varepsilon_\infty) = \frac{q^2 n}{m\omega_0^2 \varepsilon_0}, \tag{14}$$

As it follows from above, the Debye formulas are the special case of Eq. (3) solution.

Further, let's find most essential features of the relaxation polarization. In basic works [1-6] we have found a number of features. But most of them are insufficient or incorrect. We extract three doubtless features only. Below, they are cited:

1. The exponential law of the polarization decreases after a sharp cutoff of the constant external electric field. The law is expressed by the relaxation function [1]:

$$P(t) = P_0 \exp(-t/\tau), \tag{15}$$

here $P(t)$ is the polarization, P_0 is its value at the moment $t=0$.

2. The frequency dependence of relaxation polarization is described by the Debye formulas (the resonant polarization by the Lorentz formulas).

3. The relaxation function for the relaxation polarization does not include harmonic components (the resonant function includes) [1]. It is obvious that is the sequence of first feature.

Moreover, in woks [1-6] we haven't found correct the relaxation polarization determination.

3. Conclusion

The relaxation polarization model based on the linear oscillator theory can be developed. This is the main point of the study. The presented approach allows us to solve a number of the current polarization theory problems and to understand better the polarization phenomena nature. The model can be also applied to relaxation processes of various nature, such as, the magnetic phenomena, the polymers viscosity, the internal friction, etc.

Let's consider the sum of imaginary parts of functions (6) and (9):

$$\alpha''(\omega) = \frac{q^2 n}{m\omega_0^2}\left(\frac{\omega\tau}{1+\omega^2\tau^2} + \frac{1}{\omega\tau}\right). \tag{16}$$

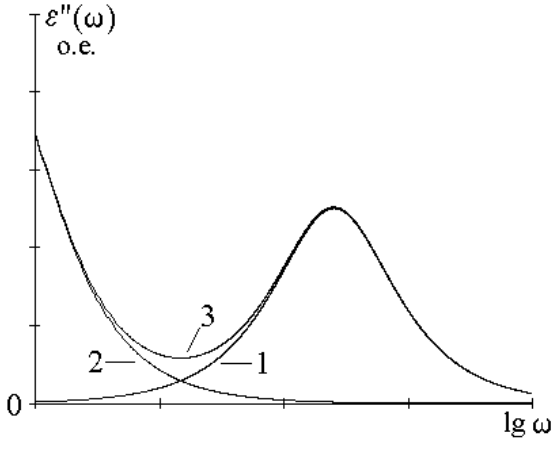

Fig. 2. 1 - energy loss for overdamped oscillator (function (6)), 2 - energy loss for strongly overdamped oscillator (function (9)), 3 - energy loss for relaxation polarization (function (16)).

As shown in Fig. 2 the graph of (16) is the typical relaxation loss spectra for the dielectric with relaxation polarization. There is the spectral formula like (16) [5, 10], but as it is considered at present, the rise of loss curve at low frequency is caused by the conductivity loss.

As it was mentioned above, there is no exact determination of relaxation polarization. As it follows from above-stated approach, the relaxation polarization is the overdamped elastic vibrations of charged particles in a dielectric under action of the external alternating field and the exponential relaxation of the polarization to constant value under action of the exter-

nal constant field. Here most essential relaxation polarization features are listed: (i) the relaxation function is exponential; (ii) the polarization frequency dependence is described by function (6) (or the Debye formulas); (iii) the vibration phase at the loss peak maximum frequency ω_m is equal to $\pi/4$ (for the resonant polarization the phase is approximately equal to $\pi/2$); (iiii) the damping is high, $\beta/\omega>3.5$.

All the cited features are interdependent. All the parameters, except damping, can be measured experimentally. Method of dielectric damping measurement is not available.

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