

SPIN-ORBIT INTERACTION AND SPINTRONICS EFFECTS IN SEMICONDUCTOR STRUCTURES

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Abstract

A review of the recent results concerning the new concepts and achievements in the area of semiconductor spintronics (or spin-electronics) is presented. Because spin-orbit interactions (SOI) couple electron spins to electric fields and allow electrical manipulation of electron spins and electrical detection of spin dynamics they form the fundamental mechanisms of new effects and the basis of different proposals for new spintronic devices. In addition to known channels of SOI in semiconductor materials and heterostructures, new terms of SOI, induced by interband interaction through the electrical polarization or atomic displacement like optical phonons are proposed and analyzed in the paper. These SOI mechanisms have the physical nature in the relativistic quantum mechanics with Lorentz boosts and this aspect is highlighted. Some particularities of the electronic states of semiconductor quantum wells (QW) related to the intrinsic Rashba SOI and Rashba like SOI induced by the electrical polarization (EP) are studied. Size quantization states are shown to transform into interface ones at some value of the in-plane QW wave vector under SOI effect. Tunneling characteristics of single barrier heterostructure are analyzed in the conditions of both types of SOIs and are established to be spin-dependent under SOI induced by the EP. Several aspects dealing with the new discovered intrinsic spin and angular momentum Hall effects (SHE and AMHE) are revealed. The SHE is investigated in a two dimensional electron system with the SOI of both intrinsic and EP induced types. New peculiarities of SHE and AMHE, induced by interband interaction of heavy and light holes are studied in p-doped semiconductors with band degeneracy in the framework of generalized Luttinger Hamiltonian. Spin-conductivity induced by the EP is shown to be proportional to difference of the inverse values of the hole wave vectors.

1. General aspects of spintronics

Spintronics is the acronym of spin-dependent electronics and it combines the intrinsic fundamental attributes of solid-state carriers – charge and spin [1]. As rapid progress in the miniaturization of semiconductor electronic devices leads toward chip features smaller than 100 nanometers in size, device engineers and physicists are inevitably faced with the looming presence of quantum mechanics - that counterintuitive and sometimes mysterious realm of physics wherein wavelike properties dominate the behavior of electrons. It is generally accepted that in the near future device dimensions are going to approach their physical limits. Figure 1 illustrates the anticipated technology nodes for the semiconductor industry development following the Semiconductor Industry Association and International Technology Roadmap for Semiconductors [2]. The dashed wall in Fig.1 is the boundary at which there are no known solutions for further development of semiconductor industry and therefore a cardinal research breakthrough is expected.

In such circumstances the new concepts and principles in electronic device architectures are a major challenge of solid-state physics. One of such concepts is the spintronic one, which is based on the quantum property of the electron known as spin, which is closely related to magnetism. Devices that rely on an electron's spin to perform their functions form the foundation of spintronics, also known as magnetoelectronics. Thus the basic concept of spintronics is the manipulation of spin currents, in contrast to mainstream electronics in which the spin of the electron is ignored. Adding the spin degree of freedom provides new effects, new capabilities and new functionalities.

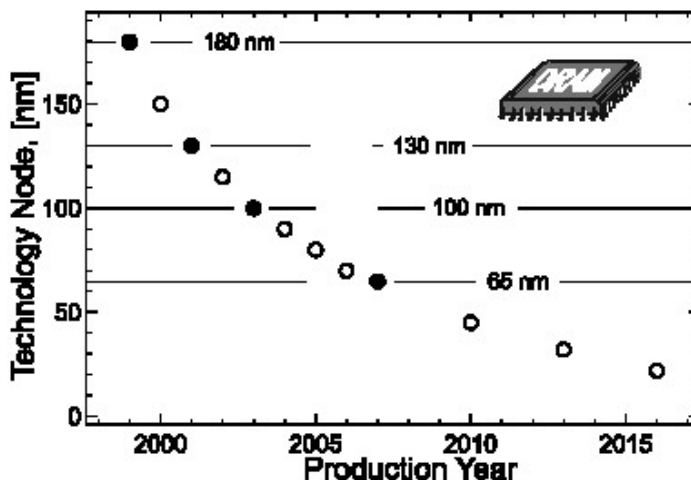


Fig. 1. Evolution of technology miniaturization for the traditional electronic devices [1].

The main goals of spintronics can be generalized in two major tasks:

- to understand the interaction between the particle spin and its solid-state environments
- to make useful devices using the acquired knowledge.

Fundamental studies of spintronics include:

- investigations of spin transport
- understanding of spin dynamics and spin relaxation.

In an ordinary electric current, the spins point at random and play no role in determining the resistance of a wire or the amplification of a transistor circuit. Spintronic devices, in contrast, rely on differences in the transport of "spin up" and "spin down" electrons. In a ferromagnetic material, such as iron or cobalt, the spins of certain electrons on neighboring atoms tend to line up. In a strongly magnetized piece of iron, this alignment extends throughout much of the metal. When a current passes through the ferromagnet, electrons of one spin direction tend to be obstructed. The result is a spin-polarized current in which all the electron spins point in the other direction. Naturally, one could try exploiting this intrinsic spin imbalance present in ferromagnetic materials to provide an extra degree of freedom to be used in semiconductor electronic devices. However, although the fundamental concepts needed to understand spin transport have been available for a longer time [3], a dramatic growth in the research in this field took place the last few years [4]. The name of spintronics is nowadays used to describe the field of research on electronic devices in which the electron spin plays the role of the active element [4]. Presently the research drive in the field of spin-dependent transport is two-fold: both commercial application and fundamental research driven. Practical spintronics incorporates existing microelectronic manufacturing techniques. It involves the development of many electronic devices that include ferromagnetic materials. It is enabling to note the development of ultra-fast switches and fully programmable all-spintronics microprocessors that can combine logic, storage and communications on a single chip. The applied side is based on the very rapid commercial success of giant and tunneling

magnetoresistance (GMR and TMR), others tunneling devices as magnetic field sensors in the read-heads of hard disks, entering large-scale production within ten years from discovery. Moreover, magnetic random access memories (MRAM) have the potential of replacing CMOS based non-volatile FLASH memories in the future [5]. On the other side, the physics of spin is interesting for the field of quantum computation [6]. The intrinsic binary nature of electronic spin suggests it could be used as the basic unit, the qubit, for quantum information storage and processing.

However, despite these practical achievements of metal magnetoelectronics semiconductor spintronics has at present the most emerging development. Why semiconductor spintronics is interesting? There are two main reasons for this

- Semiconductor-based spintronics could combine storage, detection, logic and communication capabilities on a single chip to produce a multifunctional device that could replace several components.
- Semiconductors offer a greater wealth of possibilities to create and to manipulate the nonequilibrium spin by transport, optical, and resonance methods (as well as their combination).

The basic advantages of semiconductor spintronics cover:

- long electron spin lifetime (100 ns)
- diverse types of interactions between electron spin and environment
- possibilities to tune these interactions
- controllable optical properties and the ability to amplify both optical and electrical signals

Fifteen years ago yet Datta and Das [7] have proposed a design for a spin-polarized field-effect transistor, or spin FET. In a conventional FET, a narrow semiconductor channel runs between two electrodes named the source and the drain. When voltage is applied to the gate electrode, which is above the channel, the resulting electric field drives electrons out of the channel (for instance), turning the channel into an insulator. The Datta-Das spin FET (Fig. 2) has a ferromagnetic source and drain so that the current flowing into the channel is spin-polarized. When a voltage is applied to the gate, the spins rotate as they pass through the channel and the drain rejects these anti-aligned electrons.

A spin FET would have several advantages over a conventional FET. An electron's spin flipping takes much less energy and can be done much faster than an electron pushing out of the channel. One can also imagine changing the orientation of the source or drain with a magnetic field, introducing an additional type of control that is not possible with a conventional FET: logic gates where functions can be changed on the fly.

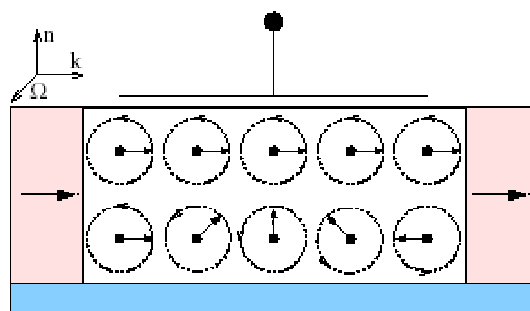


Fig. 2. Scheme of the Datta-Das spin field-effect transistor [7]

The last years many quantum structures that provide the spin-polarized states and transport were proposed and designed [1]. They can be classified in: layered magnetic structures, hybrid layered ferromagnetic/superconductor structures, hybrid ferromagnetic/semiconductor structures and pure semiconductor structures. A direct approach to obtain spin polarization of the carriers in the structure is the injection of spin-oriented carriers from the magnetic component of the structure. However for such hybrid structures it has recently pointed out a fundamental problem inherent in the electrical injection of spin currents from metal into semiconductors, the so-called conductivity mismatch. Therefore the pure non-magnetic semiconductor structures remain the more attractive systems for the spintronics.

At present fundamental studies of semiconductor spintronics include:

1. Time-resolved optical experiments
2. Spin dynamics in magnetic semiconductors
3. Spin dynamics in quantum dots
4. Spin injection
5. The Datta-Das spin transistor
6. Spin-dependent tunneling
7. Spin-related currents.
8. Nuclear spin effects in optics and electron transport.

In semiconductor spintronics the concept of spin brings an amazing new functionality into mainstream of charge-based electronics. Moreover, the spin of electron or photon can be used to store and process information locally, on the nanoscale. Though, semiconductors offer the most functional advantages (very long carrier scattering time and low noise, for example), the fabrication of semiconductor-based spintronic device and structures up to now remains a challenge despite enormous interest to this field. The lack of efficient ways to create and manipulate spin-polarized charge ensemble in a semiconductor materials or structures by electrical and optical means, spin injection, leaves semiconductor spintronics still on the level of fundamental investigations.

The purpose of this paper is to present a review of the recent results concerning the new concepts and achievements in the area of semiconductor spintronics, which has emerged last years and which includes the principles and effects of the manipulation and control of the electronic spin degree of freedom. The ability to control the electron spin requires finding the corresponding mechanisms of the spin splitting of the electronic states as well as of the spin dependent transport characteristics. The spin-orbit interactions (SOI) for carriers in semiconductor nanostructures form the basis of different proposals for new spintronic devices and this aspect is reviewed in Section 2. In particular, in addition to known channels of SOI (Rashba and Dresselhouse terms, for example) in semiconductor materials and heterostructures, new terms of SOI, induced by interband interaction through the electrical polarization or atomic displacement like optical phonons are proposed and analyzed. These SOI mechanisms have the physical nature in the relativistic quantum mechanics with Lorentz boosts and this aspect is highlighted.

In Section 3 some particularities of the electronic states of semiconductor quantum well (QW) related to the intrinsic Rashba SOI and Rashba like SOI induced by the electrical polarization (EP) are studied. The intrinsic SO coupling is shown to conserve spin degeneracy of a symmetric QW in accordance with the results of other papers. However, it influences considerably QW size quantization states transforming the latter into interface states at some

value of the in-plane wave vector. The interface states are shown to occur even in tunnel barrier structure at finite value of the in-plane wave vector. The evolution of such interface states under EP induced SOI are studied for different QWs, and it is established that they become strongly spin-splitted as well as size quantization states.

Several aspects of spin-dependent tunneling characteristics of single barrier heterostructure generated by the intrinsic Rashba SOI and Rashba like SOI induced by the electrical polarization (EP) are investigated in Section 4. In particular by the transmission coefficient and the tunneling time investigation of a symmetric barrier structure it is shown that EP induced SO coupling leads to sufficient spin-dependent tunneling characteristics of such structure.

Section 5 includes some results dealing with the new discovered recently spin Hall effect (SHE). The SHE is investigated in a two-dimensional electron system with the SOI of both intrinsic and EP induced types. A spin current perpendicular to the electric field is generated by both SOI. The direction of spin current is controllable by tuning the magnitude of the surface electric field perpendicular to the two dimensional plane via adjusting the traditional Rashba coupling and EP SOI coupling. Some new peculiarities of SHE, induced by interband interaction of heavy and light hole bands through electrical polarization or atomic displacement like optical phonon are revealed in p-doped semiconductors with band degeneracy in the framework of Luttinger Hamiltonian.

2. Spin-orbit interactions and semiconductor spintronics

In semiconductor spintronic structures the spin dynamics of carriers in general aspects can be controlled by different types of magnetic interactions - dipole-dipole interaction, spin-orbit interaction, exchange interaction, hyperfine interaction with nuclear spins. Spin-orbit interaction is considered to be the main channel for carrier spin manipulation [1,8] due to its comparatively large magnitude and due to the possibilities of using the electrical fields and optical phenomena for modification of spin states.

The spin-orbit interaction (SOI) arises as a result of the magnetic moment of the spin coupling to its orbital degree of freedom. It is actually a relativistic effect, which was first found in the emission spectra of hydrogen. SOI is a relativistic effect that occurs when a quantum mechanical particle with a non-zero spin moves in a region with a non-zero electric field. When an electron is moving with relativistic velocities in a static electric field, in the rest frame of the electron the original static electric field transforms into a field that has also a magnetic field component, its magnitude is given by the standard Lorentz transformation

$$\vec{B}_{eff}(x) = (\vec{v} \times \vec{E}(x))/c \quad (2.1)$$

The presence of this effective magnetic field that the electron feels in its rest frame affects both the dynamics of the spin and the total energy of the electron. This interaction is called spin-orbit interaction.

The Hamiltonian describing traditional SOI in quantum mechanics of a slow electron in a vacuum, derived from the four-component Dirac equation [9], has the form

$$H_{SO}(\vec{r}) = \frac{e\hbar}{4m_0c^2} \vec{\sigma}(\vec{E}(\vec{r}) \times \vec{v}) \quad (2.2)$$

where m_0 is the free electron mass, $\vec{\sigma}$ is the Pauli matrix vector and $\vec{E}(\vec{r})$ is an electric field.

Presence of the field $\vec{E}(\vec{r})$ in Eq. (1) suggests that an electron possesses a dipole moment [10]

$$\vec{P}(\vec{v}) = \frac{e\hbar}{4m_0c^2} (\vec{\sigma} \times \vec{v}) \quad (2.3)$$

Thus even for an electron in a vacuum $H_{SO}(\vec{r})$ reveals two important aspects. One is related to the fact that its magnetic moment μ of an electron is proportional to its spin $\vec{S} = \hbar\vec{\sigma}/2$ and H_{SO} is like Zeeman interactions, and the other is connected to its dipole moment, which involves electron velocity $\vec{v}$, and H_{SO} is like electro-polarization interaction.

In crystal structures due to symmetry properties of materials, band structure effects and other various factors influencing the motion of the carriers, the SOI becomes stronger and has diverse channels of manifestation [8]. The static electric field causing the spin-orbit interaction can have different physical origins, for example being the electric field of the atomic nucleus, or related to the crystal or band structure of the solid.

Enhancement of SOI in crystals occurs from two basic sources. The first part appears as a result of fast electron motion in a strong electric field near nuclei through the band structure effects. For example, the equations of the two-band scheme of narrow gap semiconductors are similar to a Dirac equation but with the forbidden gap E_g instead of the Dirac gap $2m_0c^2$. Therefore H_{SO} after the reduction to one band model becomes

$$H_{SO} = \frac{1}{2m_0} \left[\vec{p} \times \hbar \vec{\nabla} \left(\frac{V_{ext}}{2E_g} \right) \right] \vec{\sigma} \quad (2.4)$$

The difference in equations (2.2) and (2.4) reflects enormous increase in SOI in narrow gap semiconductors as compared to a vacuum.

Second type of enhancement and diversification comes from the symmetry of materials and of solid-state structures, which is essentially lower than the symmetry of a vacuum. This leads to new terms in SOI that critically change spin dynamics appearing in carrier Hamiltonians. In the area of semiconductor spintronics several types of SOI terms are considered. The specific Dresselhaus and Rashba mechanisms are more frequently explored for different spintronics applications [1,8]. The Dresselhaus SOI in bulk semiconductors [11] appears as a result of the asymmetry present in certain crystal lattices, e.g., the zinc blende structures. For a two-dimensional electron gas in semiconductor structures with appropriate growth geometry, the Dresselhaus SOI has the form [8]

$$H_D = \alpha_D (\sigma_x \hat{k}_x - \sigma_y \hat{k}_y) \quad (2.5)$$

where α_D is the Dresselhaus coupling constant, which ranges from about $2 \cdot 10^{-10}$ eV cm to $2 \cdot 10^{-9}$ eV cm and decreases rapidly with the quantum well width.

The Rashba SOI [12] arises due to the asymmetry associated with the confinement potential in z-direction of the structure and reads as

$$H_R = \alpha_R (\sigma_x \hat{k}_y - \sigma_y \hat{k}_x) \quad (2.6)$$

Being related to the structure confinement potential Rashba SOI is of interest because of the ability to electrically control the strength of this interaction.

The above mentioned mechanisms of SOI stem from the electron orbital motion and are related to the 3D spatial angular momentum. At the same time there are angular momenta related to the generators of so-called Lorentz boost, which traditionally have always been ignored because their infinitesimal generators are not conserved and their eigenvalues can not be used to label physical states [13]. However, recently [14] it was shown that the generators of Lorentz boost have a nontrivial physical significance in quantum mechanics, and they do have interactions (including SOI) related to the generators of Lorentz boost. It was shown [14] that in a charged system the nonzero space-time components of 4D angular momentum tensor (i.e. the generators of Lorentz boost) correspond to an electric moment similar as nonzero purely spatial components of 4D angular momentum tensor (i.e. the generators of 3D spatial rotation) correspond to a magnetic moment.

In this generalized case, the infinitesimal generators of the Lorentz transformation are

$$\Sigma^{\mu\nu} = \frac{i}{4} [\gamma^\mu, \gamma^\nu] \quad (2.7)$$

i.e., the 4D spin tensor of electron, which is written in (2.7) through Dirac γ matrices. Using the totally anti-symmetric tensor ε^{ijk} with $\varepsilon^{123} = 1$ (i,j,k = 1,2,3) it is easy to show that

$$\vec{\Sigma} = (\Sigma_1, \Sigma_2, \Sigma_3) \text{ with } \Sigma_i = \frac{1}{2} \varepsilon_{ijk} S^{jk}$$

is the usual spin matrices (as the generators of 3D spatial rotations). In addition to that they're appearing the generators

$$\vec{K} \equiv (S^{01}, S^{02}, S^{03})$$

of Lorentz boosts related to space-time rotations and called [14] spin-like matrices.

In terms of the Pauli's matrix vector σ , the spin matrices Σ and *spin-like* matrices K are expressed as

$$i \frac{\partial}{\partial t} \psi' = \left[\frac{(\hat{p} - e\vec{A})^2}{2m} + e\Phi - g_s \frac{e}{2m} \vec{\Sigma} \vec{B} + g_s \frac{e}{2m} \vec{K} \vec{E} \right] \psi' \quad (2.12)$$

This equation illustrates that. the electron in vacuum will behave as though it has a magnetic moment operator

$$\frac{e}{2m} \vec{\Sigma}$$

and an electric moment operator

$$\frac{e}{2m} \vec{K}$$

The magnetic moment related to the spin matrices Σ . , is nonzero in the rest frame of electron (the magnetic moment is *intrinsic*); whereas the electric moment, related to *spin-like* matrices K, is the relativistic effect of the magnetic moment (the electric moment is *induced*). Thus, just as that the spin degrees of freedom related to the infinitesimal generators of spatial rotation (i.e., the spin matrices Σ .) bring an *intrinsic* magnetic moment for electron, the *spin-like degrees of freedom* related to the infinitesimal generators of Lorentz boost (i.e., the *spin-like* matrices K) bring an *induced* electric moment for electron. The physical nature of the latter is related to the aspect that an electron with spin degrees of freedom has also the *spin-like* degrees of freedom, which belong to *the particle-antiparticle space*. This space is spanned by the positive- and negative-energy states of electron and a superposition of plane waves of positive- as well as of negative-energy is necessary to obtain a wave packet of electron. In result an *induced* electric moment for the electron occurs.

This new fundamental quantum attribute of the electron in vacuum [14] - *induced* electric moment must have a lot of richer manifestations in crystals, in particular in semiconductor materials and structure. One such new channel of interaction of the carriers in semiconductor materials and heterostructures is related to electrical polarization appearing in spontaneous way due to the atomic displacement of two sublattices of semiconductors or due to piezoelectric effect induced by strain. In the two band model of semiconductors with similar conduction and valence bands (like PbTe) a variant of such effect of the electrical polarization on the electronic states of semiconductor heterostructures has been considered in papers [15, 16]. Such term was introduced by symmetry considerations in order to describe the relative displacements of two sublattices of the semiconductor compound of lead telluride type. Being the displacement of cationic and anionic sublattices it is accompanied by electrical polarization [16]. Thus in this simplest two band model the equations of the electronic states with incorporated electrical polarization or displacement are written

$$\begin{pmatrix} h_c & h_{cv} \\ h_{vc} & h_v \end{pmatrix} \begin{pmatrix} F_c \\ F_v \end{pmatrix} = \begin{pmatrix} E_g(\vec{r})/2 + Q(\vec{r}) & \vec{\sigma} \cdot \vec{p} - i\vec{\sigma} \cdot \vec{U}(\vec{r}) \\ \vec{\sigma} \cdot \vec{p} + i\vec{\sigma} \cdot \vec{U}(\vec{r}) & -E_g(\vec{r})/2 + Q(\vec{r}) \end{pmatrix} \begin{pmatrix} F_c \\ F_v \end{pmatrix} = E \begin{pmatrix} F_c \\ F_v \end{pmatrix}, \quad (2.13)$$

where $E_g(\vec{r})$ is position dependent band gap of the semiconductor. In the case of heterojunction $Q(\vec{r})$ is the external applied potential, $\vec{p} = i\vec{p} \cdot \vec{\nabla}$ is the momentum operator ($\vec{p}$ being momentum interband matrix elements), $U(\vec{r})$ is the term related to the above-mentioned electrical polarization or displacement. F_v and F_c are the components of the spin wave function.

Equation (2.12) is of Dirac type which includes the term $\vec{\sigma} \cdot \vec{U}(\vec{r})$. Its quantum mechanical nature in connection with Lorentz boosts [14] on the one side, has been discussed above, but on the other side it represents the interaction of conduction and valence bands through optical phonons. In this aspect we mention that the interband interaction through acoustic phonons in the context of SOI and SHE has been analyzed recently [17].

In order to identify the SOI terms following traditional procedure of the transformation of Dirac-like equation in the Schrodinger ones we reduce the corresponding set of four-coupled first order differential equations. These can be written as:

$$H_c F_c = \{h_c + h_v [E - h_v]^{-1} h_{vc}\} F_c = E F_c$$

where $F_c = (f_+, f_-)$ is the conduction band envelope function spinor with the component for each spin orientation and E is the electron energy measured from the semiconductor midgap.

Following the outlined procedure and using the relation

$$(\vec{\sigma} \vec{a})(\vec{\sigma} \vec{b}) = (\vec{a} \vec{b}) + i \vec{\sigma} \vec{a} \vec{b}$$

the Hamiltonian is H_c getting the form:

$$H_c = h_c + \vec{p}(E - h_v)^{-1} \vec{p} + P \vec{\sigma} [\vec{\nabla}(E - h_v)^{-1} \times \vec{p}] + U^2 (E - h_v)^{-1} + P \vec{\nabla} [\vec{U}(E - h_v)^{-1}] - 2 \vec{\sigma} [\vec{U}(E - h_v)^{-1} \times \vec{p}] - i P \vec{\sigma} [\vec{\nabla}(E - h_v)^{-1} \times \vec{U}] - i P \vec{\sigma} \text{rot}(\vec{U}(E - h_v)^{-1}) \quad (2.15)$$

From this expression we observed that even in the two band model with included spin like degree of freedom through the mechanism of electrical polarization or atomic displacement, new SOI terms appearing in the reduced Schrodinger like Hamiltonian. The term $U^2 (E - h_v)^{-1}$ is the operator related to the energy of polarization, while the terms $P \vec{\nabla} [\vec{U}(E - h_v)^{-1}]$ express the hydrostatic and shear deformations induced by optical like displacement of the sublattices. This term is similar to the strain term that appears in the result of usual interband acoustic hydrostatic and shear deformation [17]. However in the context of the problem analyzed in the present paper the two last terms of Eq. (2.14), which describes SOI induced by electrical polarization or displacement are more important. The term $2 \vec{\sigma} [\vec{U}(E - h_v)^{-1} \times \vec{p}]$ looks like traditional SOI term $P \vec{\sigma} [\vec{\nabla}(E - h_v)^{-1} \times \vec{p}]$, where the electric field related to $\vec{\nabla}(E - h_v)^{-1}$ is replaced by electrical polarization. The last term looks more unusual. The term $i P \vec{\sigma} \text{rot}(\vec{U}(E - h_v)^{-1})$ describes the SOI induced by the vector of local rotation of the lattices $\vec{\Omega} = \frac{1}{2} \text{rot} \vec{U}$ [18] which together with symmetric tensor of deformation is the antisymmetric part of the general distortion tensor.

Also a part of analyzed term $i P \vec{\sigma} [\vec{\nabla}(E - h_v)^{-1} \times \vec{U}]$ describes the term of SOI induced by rotation like interaction of electrical and polarization fields. Following [19] and introducing the effective masses of the two band model

$$m(\vec{r}, E) = \frac{\hbar^2}{2P^2} \left(E + \frac{E_g(\vec{r})}{2} + Q(\vec{r}) \right)$$

the Hamiltonian is rewritten

$$\begin{aligned}
 H_c = & \frac{E_g(\vec{r})}{2} + Q(\vec{r}) - \frac{\hbar^2}{2} \vec{\nabla} \frac{1}{m} \vec{\nabla} + \frac{\hbar^2}{2} i \vec{\nabla} \frac{1}{m} (\vec{\sigma} \times \vec{\nabla}) + \frac{\hbar^2 U^2}{2P^2 m} + \frac{\hbar^2}{2P} \left(\vec{\nabla} \frac{\vec{U}}{m} \right) + \\
 & + \frac{i\hbar^2}{P} \vec{\sigma} \left(\frac{\vec{U}}{m} \times \vec{\nabla} \right) - \frac{i\hbar^2}{2P} \left(\vec{\sigma} \text{rot} \left[\frac{\vec{U}}{m} \right] \right)
 \end{aligned} \quad (2.16)$$

In the case of two dimensional layered heterostructure with variation of the parameters in one z direction and polarization vector $\vec{U}$ oriented in the same direction $\vec{U} = \{0, 0, U(z)\}$ the Hamiltonian H_c takes the form.

$$\begin{aligned}
 H = & -\frac{\hbar^2}{2} \frac{d}{dz} \frac{1}{m(z)} \frac{d}{dz} + \frac{\hbar^2 k_z^2}{2m(z)} + \frac{\hbar^2 U^2}{2P^2 m(z)} + \frac{\hbar^2}{2P} \frac{d}{dz} \left(\frac{U}{m(z)} \right) + \\
 & + \left[\frac{\hbar^2}{2} \frac{d}{dz} \left(\frac{1}{m(z)} \right) + \frac{\hbar^2}{P} \frac{U}{m(z)} \right] \cdot (k_x \hat{\sigma}_y - k_y \hat{\sigma}_x)
 \end{aligned} \quad (2.17)$$

where k_x and k_y are the wave vectors in the plane of the heterostructure $k_\perp^2 = k_x^2 + k_y^2$.

In this case the energy is measured from the bottom of the band. From (2.17) it follows that in this case the SOI induced by electrical polarization is Rashba like [8,12]

3. Electronic states in semiconductor quantum well structures

One of the main issues of the physics of quantum well structures in the context of spintronic investigations is the formation of spin-split states. Concerning the spin-orbit splitting of the electronic states in quantum well (QW) structures there exists a vast lot of investigations. In particular, spin splitted electronic states have been obtained in an asymmetric quantum well on the basis of Rashba mechanism and the standard envelope-function formalism starting from the eight-band Kane model for the bulk semiconductors [19]. Simpler two-band Kane model has been used later to study the spin splitting of the states in the IV-VI semiconductor QWs [20], although in the analysis of the states of symmetric variants of the QW the effect of spin-orbit coupling has been neglected. However, as will be shown below although the effect of Rashba SOI does not lead to spin splitting in the symmetric QW (as well as to spin dependent tunneling in the symmetric barrier structure) it can considerably modify the states of size quantization in QW (as well as the tunneling transmission of the barrier structure). In particular, due to Rashba SOI size quantization electronic states in the QW at finite values of the in-plane wave vectors transform into interface like states. Moreover such interface states will be shown to appear in the barrier structure yet at finite values of the in-plane wave vectors [21, 22].

The electronic states and their energy spectrum are investigated on the basis of one band Schrodinger like Hamiltonian (2.17) taking into account that reduced electrical polarization $Q(z) = U(z)/m(z)$ is oriented along axis z . The last non-diagonal term (2.17) in spin variable can be reduced to the diagonal form using the following

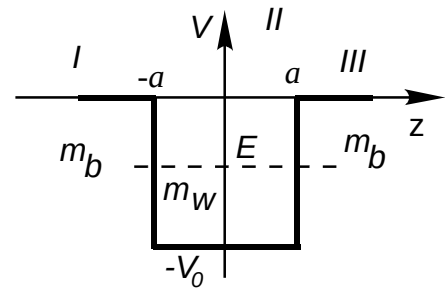


Fig. 3. The symmetric quantum well with width $2a$ and depth V_0 . Electron energy is smaller than well borders

unitary transformation

$$U = \begin{pmatrix} -e^{i\varphi} & e^{i\varphi} \\ 1 & 1 \end{pmatrix}, \quad (3.1)$$

where φ is the angle between $\vec{k}_\perp$ and the axis x . The result can be explained as $(k_x \sigma_y - k_y \sigma_x) \rightarrow k_\perp \sigma_z$

On the basis of (2.17) it is convenient for further calculation to write bulk Hamiltonian (in the region with constant mass, far from junction interface)

$$H = -\frac{\hbar^2}{2m} \nabla_z^2 + V(z) + \frac{\hbar^2}{2m} \left(k_\perp^2 + \frac{u^2}{p^2} \right) + \frac{\hbar^2 u}{2mp} k_\perp \lambda, \quad (3.2)$$

and boundary conditions such as

$$\frac{1}{m} \left[\nabla_z - \frac{u}{p} + k_\perp \lambda \right] \psi = \text{const} \quad (3.3)$$

where $\lambda = \pm 1$ is the quantum number that describes the states with spin “up” and spin “down” in the new spin-coordinate systems (3.1).

Considering potential profile for electrons as shown in Fig. 3 we can treat the problem using the Hamiltonian (3.2). The Schrodinger equation gives for each region the following wave functions

$$\psi_I = C_1 e^{q(z+a)} \quad (3.4.I)$$

$$\psi_{II} = C_2 e^{i\kappa(z+a)} + C_3 e^{-i\kappa(z+a)} \quad (3.4.II)$$

$$\psi_{III} = C_4 e^{-q(z-a)} \quad (3.4.III)$$

where

$$q = \sqrt{-\frac{2m_b E}{\hbar^2} + (k_\perp + \lambda Q_b)^2} \quad \text{is the}$$

wave vector in the barrier region and

$$\kappa = \sqrt{\frac{2m_w (E + V_0)}{\hbar^2} - (k_\perp + \lambda Q_w)^2} \quad \text{is the}$$

wave vector in the well region.

The notations $Q = \frac{U}{p}$ were used

and the index “b” notes the quantities corresponding to barrier layer (region I and III) and index “w” corresponds to well region (region II)

Using at the interface of wave functions continuity and condition (3.3) gives the relation of dispersion for size quantized electronic states [22].

$$2\gamma q \kappa \cos 2\kappa a + \{q^2 - \gamma^2 \kappa^2 - [(Q_b + k_\perp \lambda) - \gamma(Q_w + k_\perp \lambda)]^2\} \sin 2\kappa a = 0 \quad (3.5)$$

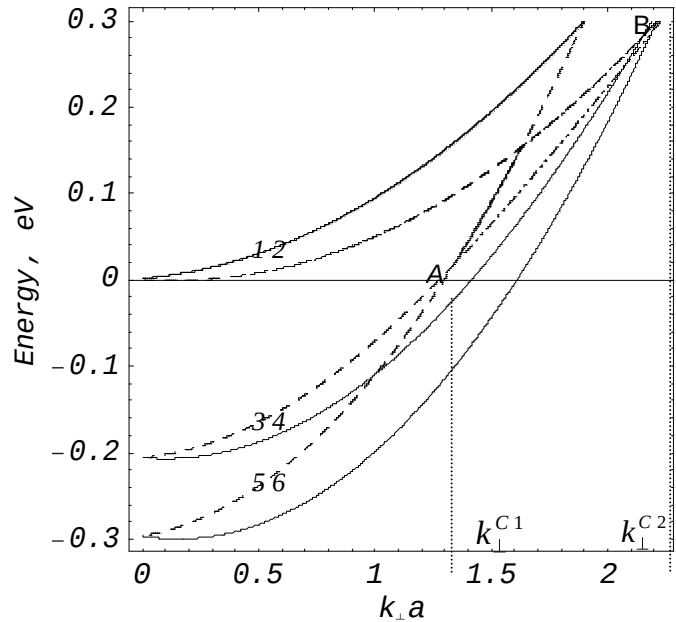


Fig. 4. Quantum well structure

1, 2, 5 and 6 are the bulk dispersion curves; 3 and 4 are an energy level for spin-up (solid line) and spin-down (dotted line) electrons.

where $\gamma = \frac{m_b}{m_w}$ is the mass rapport. It is important to outline the appearance of spin-orbit

coupling contributions in dispersion equation (3.5) for a symmetric QW without electrical polarization $Q = 0$ of the layers yet.

The interface states are described by a exponential behavior in the region I and III near the interface (no oscillatory as in the case of quantum dimensional states) and the change of wave function (3.1.II) with

$$\psi_{II} = C_2 e^{\kappa(z+a)} + C_3 e^{-\kappa(z+a)} \quad (3.6)$$

gives

$$2\gamma q \kappa \cosh 2\kappa a + \{q^2 + \gamma^2 \kappa^2 - [(Q_b + k_{\perp} \lambda) - \gamma(Q_w + k_{\perp} \lambda)]^2\} \sinh 2\kappa a = 0 \quad (3.7)$$

Relations (3.2) and (3.4) give the energy for both size quantization and interface states.

To be more explicit we present in Fig. 4 the numerical results for a quantum well with width 40 Å and depth 300 meV. The effective mass in the well structure is about 0,07 electron mass and the mass rapport is equal to 2. Electrical polarization is equal in magnitude but opposite in sign for well and its borders. Thus $Q_b = -Q_w = 4 \cdot 10^{-3} \text{ Å}^{-1}$.

The interface states for spin down electrons appear from $k_{\perp}^{C1}$ to $k_{\perp}^{C2}$ in the domain where bulk dispersions intersect (curve AB). For spin up, the situation has mirror symmetry relative to energy axis.

The discussed situation is in accordance with results obtained by Kolesnikov and Silin for single heterojunction [16, 23].

4. Spin dependent tunneling in barrier heterostructure

The new field of spintronics exploits especially the ability of conduction electrons in metals and semiconductors to carry spin-polarized current. In the context of spin injection and filtering different variants of tunnelling barrier structures have been investigated the last years. In [24, 25], in particular, an asymmetric barrier was proposed to generate spin-dependent tunnelling due to interface-induced Rashba spin-orbit coupling. The tunnelling transmission was shown to become spin-dependent in the result of spin-dependent reflection by inequivalent interfaces. Recently the Dresselhaus term in the effective Hamiltonian of bulk semiconductor of symmetric barrier was shown to result in a dependence of the tunnelling transmission on the carrier spin orientation [26]. However, the difference of the transmission probability for opposite spin orientations can achieve several percents as well as in the structures with Rashba mechanism of spin polarisation. In order to illustrate the possibilities of combined intrinsic SOI and extrinsic SOI, induced by the electrical polarization, to form spin polarized tunnel current a simple barrier semiconductor structure is analyzed taking in account only the conduction band, variation of the potential energy in the direction normal to the barrier is shown in fig. 3.

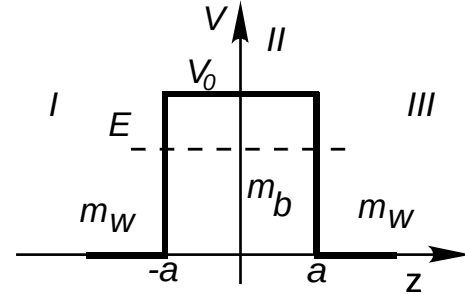


Fig. 5. Variation of the potential energy in the direction normal to the barrier. The potential energy is zero in regions I and III.

Solutions of one-dimensional Schrödinger equation for each region with Hamiltonian given by (3.2) read

$$\psi_I = C_1 e^{iq(z+a)} + C_2 e^{-iq(z+a)} \quad (4.1.1)$$

$$\psi_{II} = C_3 e^{\kappa(z+a)} + C_4 e^{-\kappa(z+a)} \quad (4.1.2)$$

$$\psi_{III} = C_5 e^{-iq(z-a)} \quad (4.1.3)$$

where $q = \sqrt{\frac{2m_w E}{\hbar^2} - (k_{\perp} + \lambda Q_w)^2}$ and $\kappa = \sqrt{\frac{2m_b(V_0 - E)}{\hbar^2} + (k_{\perp} + \lambda Q_b)^2}$ are the wave vectors in the well and in the barrier.

After application of boundary conditions presented in section 2 the relation between emergent and incident amplitudes of electron beam takes the form

$$\frac{C_5}{C_1} = G \exp[i\alpha] \quad (4.2)$$

The transmission coefficient T_C and the tunnelling time T_T are given in terms of G and α by the relations [27]

$$T_C = GG^* \quad (4.3)$$

and

$$T_T = \frac{m_w}{\hbar q} \frac{\partial \alpha}{\partial q} \quad (4.4)$$

Explicit relations for the mentioned quantities are

$$T_C = \frac{B^2}{A^2 \sinh^2 2\kappa a + B^2 \cosh^2 2\kappa a} \quad (4.5)$$

$$T_T = \frac{m_w}{\hbar q \kappa} \frac{\left[\frac{1}{2} B^2 (\gamma + 1) + A \gamma (\gamma^2 q^2 - \kappa^2) \right] \sinh 4a\kappa - 4aA\gamma^2 q^2 \kappa}{A^2 \sinh^2 2\kappa a + B^2 \cosh^2 2\kappa a}, \quad (4.6)$$

where

$$A = \gamma^2 q^2 - \kappa^2 + [\gamma(Q_w + \lambda k_{\perp}) - (Q_b + \lambda k_{\perp})]^2 \text{ and } B = 2\gamma\kappa q$$

Relations (4.5) and (4.6) correspond to relations obtained by Paranjape [27] for $Q = 0$, and $\lambda = 0$ i.e. in the absence of polarization for spinless electrons.

The important characteristics for tunneling of electron with spin are the polarizability

$$P = \frac{T_C^+ - T_C^-}{T_C^+ + T_C^-} \quad (4.7)$$

and tunneling current

$$I^{\pm} = \frac{2}{\pi} \int_0^{\pi/2} T_C^{\pm} d\theta \quad (4.8)$$

where $\pm$ means the quantities for spin up or spin down ($\lambda = 1$ or $\lambda = -1$)

Relations (4.5) – (4.8) take into account the space dependent electron effective mass, the motion in the perpendicular and parallel direction to the barrier, the spin of incident electrons and the electrical polarization due to the stress of layers. We have plotted the transmission coefficient via angle of incidence (see Fig. 6) for a symmetric barrier with height $V_0 = 300 \text{ meV}$ and width $2a = 40 \text{ \AA}$ for different energy of incident electrons. The effective mass of incident electrons is $0,067 m_0$ and mass ratio $\gamma = 2$. From graphic it is clear that there is a great difference for transmission probability of electrons with up or down spin orientation. This fact can be used to obtain a spin polarization current. Although the last three terms in expression (2.17) modify potential (decreasing the height of the barrier and increasing the levels of the borders) the curves B, C and D presented in Fig. 6 plot the under barrier transmission and curve A is for on barrier electrons.

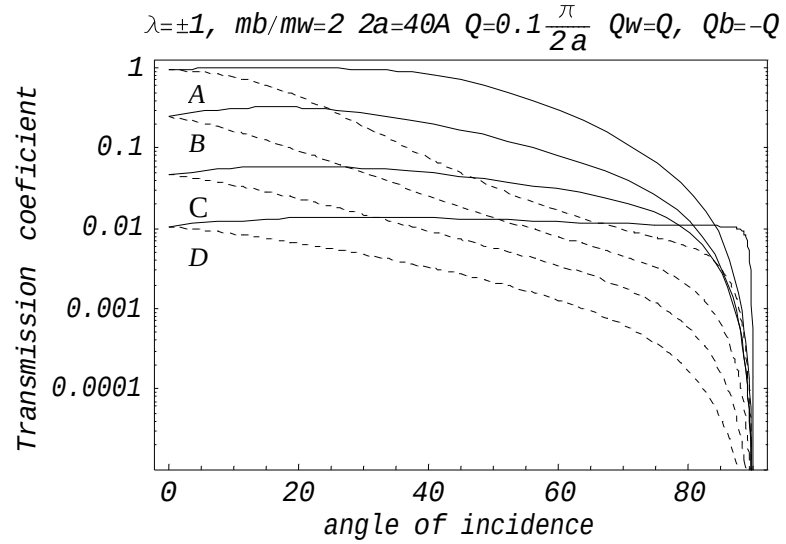


Fig. 6. Variation of the transmission coefficient via angle of incidence of electron beam for barrier structure with height $V_0 = 300 \text{ meV}$ and width $2a = 40 \text{ \AA}$. Curves A, B, C and D are plotted for incident energy $E = 450, 350, 250, 150 \text{ meV}$ respectively. Dotted lines are for spin down and solid lines for spin up.

5. Spin and orbital momentum Hall effect

In the area of emerging field of semiconductor spintronics one of the difficult obstacles is an efficient spin injection into semiconductors. One way is to make last ferromagnetic, such as (Ga, Mn)As [28]. However its Curie temperature is still lower than the room temperature, and there are still rooms for improvement towards practical use. Among other variants of spin injection during the last years spin Hall effect (SHE) was analyzed, which in its extrinsic form has been discovered more than two decades ago [29]. The extrinsic SHE is related to skew-scattering mechanism [29] and is based on asymmetric scattering of spin-up and spin-down electrons from a random impurity potential that includes a SOI term. However, the real practical interest to this approach of spin current generation has appeared recently after the discovery of intrinsic SHE [3h,4h], when Murakami, Nagaosa, and Zhang [30] proposed it in hole doped semiconductors like p-GaAs, and Sinova et al. [31] proposed the SHE in n-type two-dimensional semiconductor heterostructures. After this, two seminal experiments on the SHE have been done. Kato et al. [32] observed spin accumulation in n-type GaAs by means

of Kerr rotation, and Wunderlich et al. [33] observed a circularly polarized light emitted from a light-emitting diode (LED) structure, confirming the SHE in p-type semiconductors.

The geometrical structure of the intrinsic SHE is such that for an electric field applied, for example, on the z direction, a y -polarized dissipationless spin current will flow in the x direction (Fig.7). The electric field induced spin current can be summarized by the following macroscopic relation

$$J_j^i = \sigma_S \varepsilon_{ijk} E_k \quad (5.1)$$

Where σ_S is the spin conductance, ε_{ijk} is antisymmetric tensor.

At the same time, at the microscopic level the spin current is defined by the velocity times the spin, which is a rank two tensor. However, the velocity operator in general does not commute with the spin operator in a model with spin-orbit coupling. In order to define the spin current tensor as Hermitian operator that is symmetrized as:

$$J_j^i = \frac{1}{2} \left(S_i \frac{\partial H}{\partial P_j} + \frac{\partial H}{\partial P_j} S_i \right) \quad (5.2)$$

where S_i is the projection of the spin.

The induced spin current is considered to be dissipationless and unlike the anomalous Hall effect can be realized even in non-magnetic systems. Murakami et al.[30] and Sinova et al [31]. have argued, in contexts of p-doped semiconductors and 2D Rashba coupled systems, that spin-Hall conductivity can be dominated by an intrinsic contribution that follows from distortion of Bloch electrons by the electric field and therefore approaches a disorder independent value in high mobility systems. The aspect of dissipationless is one of the central issues of SHE both from fundamental and practical point of view. Although papers [30,31] dedicated to the topics of intrinsic SHE have appeared just at the same time

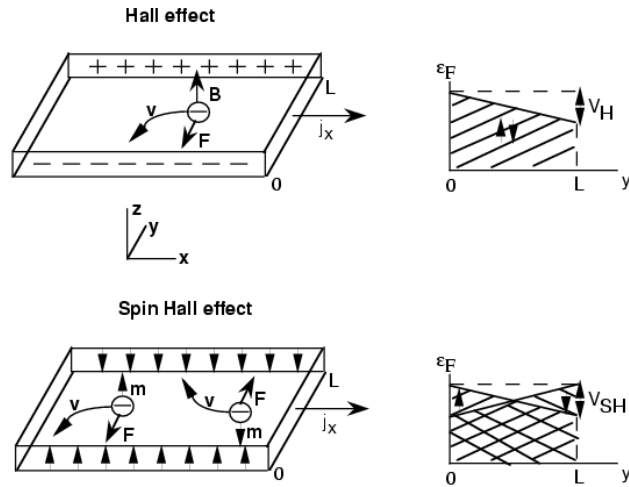


Fig. 7. Schematic illustration of the traditional Hall effect and spin Hall effect geometries with the evolution of Fermi level

In the first papers the analysis of intrinsic SHE was done on the basis of semiclassical approach. In two-dimensional n-type systems, studied in [31], the Hall conductivity σ_{xy} in a clean limit is calculated from the Kubo formula as

$$\sigma_{xy} = -\frac{e^2}{2\pi\hbar} \sum_n \int_{BZ} d^2\vec{k} n_F(\varepsilon_n(\vec{k})) B_{nz}(\vec{k}) \quad (5.3)$$

where n is the band index, and integral is over the entire Brillouin zone. $B_{nz}(\vec{k})$ is defined as a z component of $\vec{B}_n(\vec{k}) = \nabla_{\vec{k}} \vec{A}_n(\vec{k})$, where

$$\vec{A}_n(\vec{k}) = -i \left\langle nk \left| \frac{\partial}{\partial k_i} \right| nk \right\rangle \equiv -i \int_{\text{unit cell}} u_{nk}^+ \frac{\partial u_{nk}}{\partial k_i} d^2x \quad (5.4)$$

and u_{nk} is the periodic part of the Bloch wave function $\phi_{nk}(x) = e^{ikx} u_{nk}(x)$. This $B_{nz}(\vec{k})$ represents the effect of Berry phase momentum space. $n_F(\varepsilon_n(\vec{k}))$ is the Fermi distribution function for the n -th band.

By incorporating the effect $B(k)$, the Boltzman-type semiclassical equation of motion (SEOM) acquires an additional term [34]:

$$\dot{x} = \frac{1}{\hbar} \frac{\partial E_n(k)}{\partial k} + \dot{k} \times B_n(k) \quad \hbar \dot{k} = -e(E + \dot{x} \times B(x)) \quad (5.5)$$

The term $\dot{x} \times B(x)$ represents the effect of Berry phase, and it is called an anomalous velocity. Under the external electric field, the anomalous velocity becomes perpendicular to the field, and gives rise to the Hall effect. This Hall current is distinct from the usual Ohmic current, which comes from the shift of the Fermi surface from its equilibrium. This Hall effect comes from all the occupied states, not only from the states on the Fermi level. Sinova et al. [31] applied the Kubo formula to this Rashba Hamiltonian (2.6). For this procedure, they defined the spin current J_{zy} to be a symmetrized product of the spin S_z and the velocity

$$v_y = \frac{\partial H}{\partial k_y}. \quad (5.6)$$

By assuming no disorder, the resulting spin Hall conductivity

$$\sigma_{sH} = \frac{-e}{16\pi\lambda m} (p_{F+} - p_{F-}) \quad (5.7)$$

where p_{F+} and p_{F-} are the Fermi momenta of the majority and minority spin Rashba bands. However, when $n_{2D} > m^2\lambda^2/\pi\hbar^4 \equiv n_{2D}^*$, $p_{F+} - p_{F-} = 2m\lambda/\hbar$ and the spin Hall conductivity is

$$\sigma_{sH} = -\frac{j_{sy}}{E_z} = \frac{e}{8\pi} \quad (5.8)$$

independent of both the Rashba coupling strength and the 2DES density.

The origin of the SHE in this model is the following. The Rashba term (2.6) can be regarded as a k -dependent effective Zeeman field $B_{eff} = \alpha(z \times k)$. In equilibrium the spins are pointing either parallel or antiparallel to B_{eff} for the lower and upper bands, respectively. An external electric field $E\parallel x$ changes the wave vectors k of Bloch wave functions, and B_{eff} also changes accordingly. The spins will then precess around B_{eff} , and tilt to the $\pm z$ -direction, depending on the sign of k_y . This appears as the SHE, and the spin Hall

conductivity is calculated to be $\frac{e}{8\pi}$, in agreement with the Kubo formula. The Dresselhaus term (2.5) can also be incorporated and if this SOI is stronger than Rashba ones spin Hall conductivity changes the sign.

By straightforward calculation following [31] it is easy to incorporate the influence of Rashba like SOI induced by electrical polarization, oriented normal to the 2D systems. As follows from (2.17) in this case the electrical induced SOI leads only to renormalization of Rashba constant.

In the general case both Rashba and Dresselhaus like terms are generated by electrical polarization appearing, and its effect is similar to established effect of intrinsic ones.

Initially the intrinsic SHE in p-type systems has been studied [30] in bulk semiconductors on the basis of isotropic effective Luttinger Hamiltonian of the two-fold degenerate valence bands of conventional semiconductors such as Si, Ge, GaAs et al

$$H_L = \frac{1}{2m} \left[\left(\gamma_1 + \frac{5}{2} \gamma_2 \right) P^2 - 2\gamma_2 (PS)^2 \right] \quad (5.9)$$

where $S = (S_x, S_y, S_z)$ are spin 3/2 matrices, γ_1 and γ_2 are the Luttinger constants.

Following [35] in semiclassical approximation is straightforward to get the relation for spin Hall conductivity. The semiclassical equations of motion for holes under an electric field E read as

$$\dot{\mathbf{k}} = \frac{1}{\hbar} \frac{\partial E_\lambda(\mathbf{k})}{\partial \mathbf{k}} + \dot{\mathbf{k}} \times \mathbf{B}_\lambda(\mathbf{k}) \quad \hbar \dot{\mathbf{k}} = -e\mathbf{E} \quad (5.10)$$

Here $E_\lambda(\mathbf{k}) = \frac{\hbar^2 \mathbf{k}^2}{2m} \left(\gamma_1 + \left(\frac{5}{2} - 2\lambda^2 \right) \gamma_2 \right)$ is an eigenenergy for the band with helicity

λ . The Berry curvature is given by

$B_\lambda(\mathbf{k}) = \lambda(2\lambda^2 - 7/2) \bar{\mathbf{k}}/k^3$, which depends on the helicity.

If there is no anomalous velocity, the holes will move parallel to $\bar{\mathbf{k}}$. The Berry curvature gives rise to an anomalous velocity perpendicular to both $\bar{\mathbf{k}}$ and $\bar{\mathbf{E}}$. This anomalous velocity is opposite for opposite helicity, i.e. for opposite

spin orientation. This anomalous velocity from the motion along $\bar{\mathbf{k}}$, when summed over all $\bar{\mathbf{k}}$ of occupied states, amounts to a spin current. At the zero temperature, by applying an electric

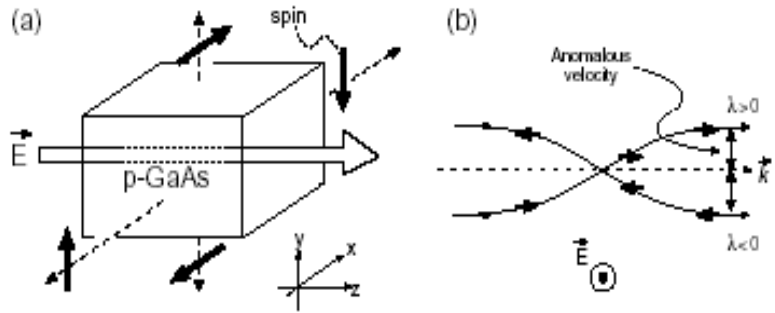


Fig. 8

(a) Schematic of the spin Hall effect for p-type semiconductors [35].

(b) Semiclassical trajectories for the wavepackets with helicity λ , projected onto the plane perpendicular to the electric field E . The thick arrows represent the spins.

field along the l-direction, the spin current in which spins along the i-axis will flow along the j-direction is calculated as

$$j_j^i = \frac{e}{12\pi^2} (3k_F^H - k_F^L) \varepsilon_{ijl} E_l \quad (5.11)$$

Here, k_F^H and k_F^L are the Fermi wavenumbers for the HH and the LH bands, respectively.

A schematic of this effect is shown in Fig. 8 (a).

Taking into account the interband interaction through optical phonons [36] the SOI induced by electrical polarization or optical like displacement can be incorporated into effective Luttinger Hamiltonian in the following form

$$\begin{pmatrix} 0 & iu_y + u_x & -iu_z & 0 \\ -iu_y + u_x & 0 & 0 & -iu_z \\ iu_z & 0 & 0 & -iu_y - u_x \\ 0 & iu_z & iu_y - u_x & 0 \end{pmatrix} \quad (5.12)$$

Thus the generalized effective Luttinger Hamiltonian including both induced SOI and the anisotropy terms of the holes is written as

$$\hat{H} = \frac{\hbar^2}{2m^*} \left[\left(\gamma_1 + \frac{5}{2} \gamma_2 \right) k^2 - 2\gamma_2 (k_x^2 S_x^2 + k_y^2 S_y^2 + k_z^2 S_z^2) - \right. \\ \left. - 4\gamma_3 (\{k_x, k_y + u_z\} \{S_x, S_y\} + \{k_y, k_z + u_x\} \{S_y, S_z\} + \{k_z, k_x + u_y\} \{S_z, S_x\}) \right] \quad (5.13)$$

where $[S_i, S_j] = \frac{1}{2} (S_i S_j - S_j S_i)$.

Taking into account that Luttinger like Hamiltonian (1) has two eigenvalues of light and heavy hole states following the recently proposed approach [37] terms in the anisotropic Luttinger Hamiltonian can be transformed in terms of a Clifford algebra of Dirac Γ matrices

$$\{\Gamma_a, \Gamma_b\} = a\delta_{ab} I_{4 \times 4} \quad \hat{H} = \gamma_1 k^2 + 2\gamma_3 d_a \Gamma_a$$

Where

$$\begin{aligned} d_1 &= -\sqrt{3}k_z k_y + \sqrt{3}u_x \\ d_2 &= -\sqrt{3}k_z k_x + \sqrt{3}u_y \\ d_3 &= -\sqrt{3}k_x k_y + \sqrt{3}u_z \\ d_4 &= -\frac{\sqrt{3}}{2} \frac{\gamma_2}{\gamma_3} (k_x^2 - k_y^2) \\ d_5 &= -\frac{1}{2} \frac{\gamma_2}{\gamma_3} (2k_z^2 - k_x^2 - k_y^2) \\ \text{and } \frac{\hbar^2}{2m^*} \gamma_j &\rightarrow \gamma_j \end{aligned}$$

The eigenvalue of the operator $d_a \Gamma / d$, where $d^2 = d_a d_a$ are ± 1 so that, the eigenvalues of $\hat{H}$ are:

$$E(k) = \gamma_1 k^2 \pm 2\gamma_3 d(k)$$

where

$$d^2(k) = \left(\frac{\gamma_2}{\gamma_3}\right)^2 k^4 + 3 \left(1 - \left(\frac{\gamma_2}{\gamma_1}\right)^2\right) (k_x^2 k_y^2 + k_y^2 k_z^2 + k_z^2 k_x^2) + 3u^2 + 6(k_x k_y u_z - k_y k_z u_x - k_z k_x u_y)$$

The terms with polarization SOI lead to the spin splitting of heavy and light holes as illustrated in the Fig, 9,

Following the approach [37] in spherical approximation and keeping only terms linear in u_j the expression for spin conductance is obtained: in analytical form:

$$\sigma_{ij}^l = \sigma_{ij}^l(0) + \sigma_{ij}^l(u) = \frac{1}{6\pi^2} \varepsilon_{ijl} (k_F^H - k_F^L) + \frac{1}{6\pi^2} \varepsilon_{ijl} u_l \left(\frac{1}{k_F^L} - \frac{1}{k_F^H} \right) \quad (5.14)$$

where k_F^H и k_F^L are the Fermi wave vector of heavy and light holes of bulk semiconductors respectively, ε_{ijl} is the antisymmetric tensor.

The obtained formula (5.14) for spin-conductivity shows that the part $\sigma_{ij}^l(u)$ of conductivity induced by the polarization u in comparison with intrinsic part $\sigma_{ij}^l(0)$, considered in [30] is proportional to difference of the inverse values of the hole wave vectors. In result $\sigma_{ij}^l(u)$ for low doped semiconductors can exceed $\sigma_{ij}^l(0) \cdot \sigma_{ij}^l(0)$.

The induced spin currents in the SHE are considered to be dissipationless. Murakami et al.[31] in the first paper have argued, that spin current approaches a disorder independent value in high mobility systems. The aspect of dissipationless is one of the central issues of SHE as well as recently proposed orbital angular momentum Hall effect (OAHE) from both fundamental and practical points of view. Although first papers [30,31], dedicated to the topics of intrinsic SHE, have appeared just at the same time, the proposed mechanisms of SHE are of different nature as follows from the above short analysis. In the Luttinger spin orbit coupling systems, a beautiful physical interpretation of the nature of intrinsic SHE was done. The origin of the spin- current comes from the existence of a monopole structure in momentum space, which is directly derived from the Berry phase in spin orbit coupling systems when Hamiltonian is projected to double degenerated helicity bands. Due to this topological nature, the conserved spin Hall current in general can be called as topological spin Hall current.

Recently [38], an orbital angular momentum Hall effect was proposed for two-dimensional systems with both Rashba and Dresselhaus SOI. It was demonstrated, that the orbital Hall current is different from the spin Hall current [31] and has a topological origin related to the existence of the magnetic flux in momentum space, while there is no such topological nature for the spin Hall current. In this context it is argued that the orbital Hall

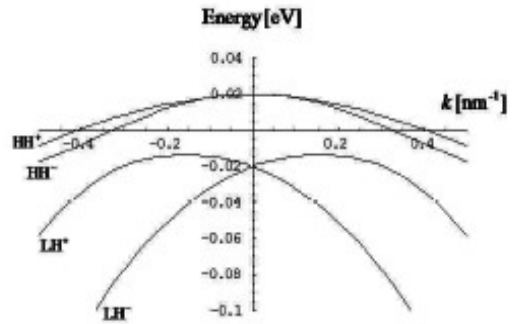


Fig.. 9 Band splitting by optical phonon like interband coupling in hole semiconductors.

current dominates the transport properties in such systems, especially in disordered limit. The amprints of SHE are suggested to be only measurable in the ballistic regime.

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