NORMALIZED REPRESENTATION OF THE EQUATIONS OF ACTIVE MULTIPORT NETWORKS ON THE BASIS OF PROJECTIVE GEOMETRY

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Abstract

To interpret the mutual influence of loads of active multiport networks, some methods of projective geometry are used. The application of the projective coordinates gives the possibility to derive an equation of the network in a normalized or relative form as well as to define the scales for the currents and conductivity of the loads. This approach makes it possible to identify parameters of the regime and to compare the regimes of different multiports. The given approach is applicable to the analysis of «flowed» form processes of different physical nature. The obtained results can be useful in the educational purposes. This new mathematical apparatus for the circuit theory is widely applied in other areas of science.

1. Introduction

The definition of parameters of an operating regime in a normalized form with respect to some characteristic values (as scales) is important for electric circuits with changeable parameters of loads. In practice, in power supply systems (for example, direct current), the load regime can vary largely from the generation to the regeneration of energy. Therefore, the definition of regimes (hereinafter referred to as relative regimes) allows comparing or setting the regimes of different systems. For example, for a circuit in the form of an active two-pole, open circuit voltage, short-circuit current, and internal resistance [1] can be the scales for the voltage, current, and resistance of load. Then, the normalized equation of the circuit (load straight line) is obtained evidently. Therefore, the regimes of compared active two poles will be identical, if the normalized values of parameters of loads are also identical. However, the consideration of more complex circuits reveals no triviality of the problem, because a number of characteristic values are increased and well-founded approaches to the formation of relative expressions are missing. For example, suppose that any resistance is changed in the active two-pole. Then, the open circuit voltage is also changed. This change in the scale of the load voltage complicates the estimation of the regime of the circuit, creates uncertainty for comparison of the regimes of the different circuits. Further, consider an active two-port network with changeable loads. Therefore, an interference of changes in the load resistances on the value of voltage of these loads takes place. This brings up the problem of choice of the scales for the voltages or currents of loads and the problem of the normalized formulation of the circuit equation.

In a number of papers of the author, the approach is developed for interpretation of changes or "kinematics" of the circuit regimes on the basis of projective geometry [2]. The formulated theorem of a generalized equivalent generator of an active two-pole develops the well-known

Kirchhoff-Thevenin and Norton theorems. It appears that the load straight line is transformed into a bunch of lines for various values of the internal resistance of this two-pole network. Since the coordinates of the center of the bunch do not depend on this changeable element, they can be accepted already as the parameters of the generalized equivalent generator to obtain the normalized parameters of the regime and normalized equation of active two-ports [3].

In this work, the results are applied to active multiport networks.

2. Projective coordinates of an active multiport network

Let us use the approach for an active two-port network with changeable conductivities of loads [3]. Now consider the active multiport network in Fig. 1. Taking into account the specified directions of currents, this network is described by the following system of the equations:

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -Y_{11} & Y_{12} & Y_{13} \\ Y_{12} & -Y_{22} & Y_{23} \\ Y_{13} & Y_{23} & -Y_{33} \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} + \begin{pmatrix} I_1^{SC} \\ I_2^{SC} \\ I_3^{SC} \end{pmatrix}, \tag{1}$$

where $Y_{11} = y_1 + y_{14} - \dfrac{y_{14}^2}{y_\Sigma}$, $y_\Sigma = y_{04} + y_{14} + y_{24} + y_{34}$, $Y_{12} = y_{24}\dfrac{y_{14}}{y_\Sigma}$, $Y_{13} = y_{34}\dfrac{y_{14}}{y_\Sigma}$,

$Y_{22} = y_2 + y_{24} - \dfrac{y_{24}^2}{y_\Sigma}$, $Y_{23} = y_{34}\dfrac{y_{24}}{y_\Sigma}$, $Y_{33} = y_3 + y_{34} - \dfrac{y_{34}^2}{y_\Sigma}$,

The short-circuit (SC) currents of all loads are

$I_1^{SC} = Y_{10}V_0 = y_{04}\dfrac{y_{14}}{y_\Sigma}V_0$, $I_2^{SC} = Y_{20}V_0 = y_{04}\dfrac{y_{24}}{y_\Sigma}V_0$, $I_3^{SC} = Y_{30}V_0 = y_{04}\dfrac{y_{34}}{y_\Sigma}V_0$.

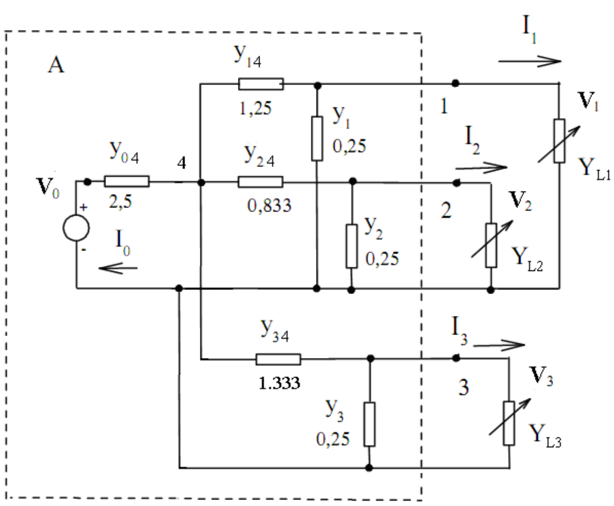

Fig. 1. Active multiport network

Taking into account the voltages $V_1 = I_1 / Y_{L1}, V_2 = I_2 / Y_{L2}$, $V_3 = I_2 / Y_{L3}$, the equations of three bunches of planes are obtained from the system of the equations (1), similarly to the bunches of straight lines of an active two-port. For visualization, two bunches of these straight lines are presented in Fig. 2.

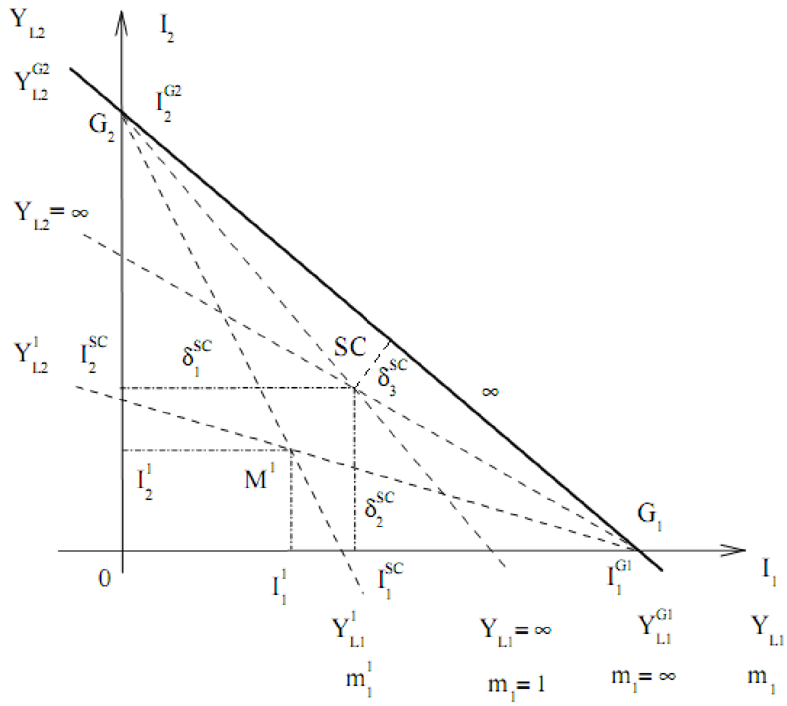

Fig. 2. Two bunches of straight lines with parameters Y_{L1}, Y_{L2}

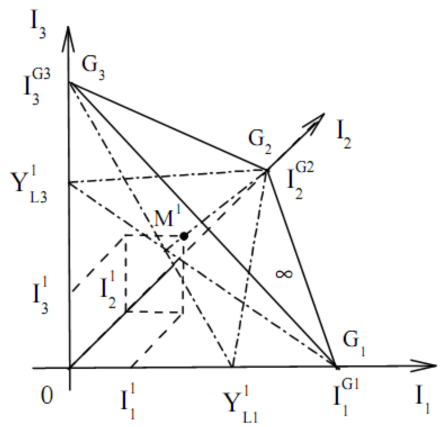

Fig. 3. Two bunches of planes with parameters Y_{L1}, Y_{L3}

The intersection of the planes of one bunch defines the bunch axis. The equation of the axis of the bunch Y_{L1} corresponds to the condition $I_1 = 0, V_1 = 0$. Therefore, this axis is located in the plane I_2, I_3, as it is shown in Fig. 3.

The point of intersection with the current axis I_2 is defined by the expression

$$I_2^{G2} = V_0\, y_{04}\left(1 + \frac{y_2}{y_{24}}\right) \tag{2}$$

that corresponds to voltage and conductivity of the second load:

$$V_2^{G2} = -\frac{Y_{10}}{Y_{12}} V_0 = -\frac{y_{04}}{y_{24}} V_0,\ \ Y_{L2}^{G2} = -(y_{24} + y_2).$$

The point of intersection with the current axis I_3 defines the third load:

$$I_3^{G3} = V_0\, y_{04}\left(1 + \frac{y_3}{y_{34}}\right),\ V_3^{G3} = -\frac{y_{04}}{y_{34}} V_0,\ Y_{L3}^{G3} = -(y_{34} + y_3) \tag{3}$$

Similarly, the axis of the bunch Y_{L2} corresponds to the condition $I_2 = 0$ and is located in the plane I_1, I_3. The point of intersection with the current axis I_1 defines the first load:

$$I_1^{G1} = V_0\, y_{04}\left(1 + \frac{y_1}{y_{14}}\right),\ V_1^{G1} = -\frac{y_{04}}{y_{14}} V_0,\ \ Y_{L1}^{G1} = -(y_{14} + y_1) \tag{4}$$

The characteristic regime is the short-circuit regime of the loads ($Y_{L1} = \infty, Y_{L2} = \infty, Y_{L3} = \infty$), which is presented by the point SC in Fig. 2. The open circuit regime of the loads is also characteristic and corresponds to the origin of coordinates, point 0.

In addition, we regard the plane which passes through three points $I_1^{G1}, I_2^{G2}, I_3^{G3}$ on the axes of coordinates as the plane of infinity ∞. This plane and three coordinate planes $I_1 = 0, I_2 = 0, I_3 = 0$ form the coordinate tetrahedron $0\,G_1 G_2 G_3$. Let the initial or running regime correspond to the point M^1 which is set by the values of conductivities $Y_{L1}^1,\ Y_{L2}^1, Y_{L3}^1$ and currents $I_1^1,\ I_2^1, I_3^1$. This point is defined by projective non-uniform coordinates m_1^1, m_2^1, m_3^1 and homogeneous $\xi_1^1, \xi_2^1, \xi_3^1, \xi_4^1$ coordinates which are set by the coordinate tetrahedron $0\,G_1 G_2 G_3$ and the unit point SC [4, 5]. Then, the homogeneous coordinates are defined as the ratio of distances of points M^1, SC to the planes of the coordinate tetrahedron:

$$\rho\xi_1^1 = \frac{I_1^1}{I_1^{SC}}\ , \quad \rho\xi_2^1 = \frac{I_2^1}{I_2^{SC}}\ , \quad \rho\xi_3^1 = \frac{I_3^1}{I_3^{SC}}\ , \quad \rho\xi_4^1 = \frac{\delta_4^1}{\delta_4^{SC}} \tag{5}$$

Distances:

$$\delta_4^1 = \frac{1}{\mu_4}\left(\frac{I_1^1}{I_1^{G1}} + \frac{I_2^1}{I_2^{G2}} + \frac{I_3^1}{I_3^{G2}} - 1\right),\ \delta_4^{SC} = \frac{1}{\mu_4}\left(\frac{I_1^{SC}}{I_1^{G1}} + \frac{I_2^{SC}}{I_2^{G2}} + \frac{I_3^{SC}}{I_3^{G2}} - 1\right),$$

where $\mu_4 = \sqrt{\frac{1}{\left(I_1^{G1}\right)^2} + \frac{1}{\left(I_2^{G2}\right)^2} + \frac{1}{\left(I_3^{G3}\right)^2}}$ is a normalizing factor.

The non-uniform projective coordinate m_1^1 is set by a cross ratio of four points:

$$m_1^1 = (0\ Y_{L1}^1\ \infty\ Y_{L1}^{G1}\ ) = \frac{Y_{L1}^1}{Y_{L1}^1 - Y_{L1}^{G1}} \div \frac{\infty - 0}{\infty - Y_{L1}^{G1}} = \frac{Y_{L1}^1}{Y_{L1}^1 - Y_{L1}^{G1}}\ . \tag{6}$$

The points $Y_{L1} = 0,\ Y_{L1}^1 = Y_{L1}^{G1}$ correspond to extreme or base values. The point $Y_{L1} = \infty$ is the unit point. The values of m_1 are also shown in Fig. 2. For the point $Y_{L1}^1 = Y_{L1}^{G1}$, the coordinate $m_1 = \infty$ defines the sense of line of infinity $G_1\ G_2$. The cross ratio for m_2^1, m_3^1 is expressed similarly.

In addition, the homogeneous projective coordinates ξ_1 , ξ_2 , ξ_3 set the non-uniform coordinates as follows:

$$m_1 = \frac{\xi_1}{\xi_4} = \frac{\rho\xi_1}{\rho\xi_4},\ m_2 = \frac{\xi_2}{\xi_4} = \frac{\rho\xi_2}{\rho\xi_4},\quad m_3 = \frac{\xi_3}{\xi_4} = \frac{\rho\xi_3}{\rho\xi_4} \tag{7}$$

where ρ is the coefficient of proportionality.

The homogeneous projective coordinates (5) have a matrix form:

$$\begin{pmatrix} \rho\xi_1 \\ \rho\xi_2 \\ \rho\xi_3 \\ \rho\xi_4 \end{pmatrix} = \begin{pmatrix} \frac{1}{I_1^{SC}} & 0 & 0 & 0 \\ 0 & \frac{1}{I_2^{SC}} & 0 & 0 \\ 0 & 0 & \frac{1}{I_3^{SC}} & 0 \\ \frac{1}{I_1^{G1}\mu_4\delta_4^{SC}} & \frac{1}{I_2^{G2}\mu_4\delta_4^{SC}} & \frac{1}{I_3^{G3}\mu_4\delta_4^{SC}} & -\frac{1}{\mu_4\delta_4^{SC}} \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ 1 \end{pmatrix} = [C]\cdot[I] \tag{8}$$

The inverse transformation:

$$\begin{pmatrix} \rho I_1 \\ \rho I_2 \\ \rho I_3 \\ \rho 1 \end{pmatrix} = \begin{pmatrix} I_1^{SC} & 0 & 0 & 0 \\ 0 & I_2^{SC} & 0 & 0 \\ 0 & 0 & I_3^{SC} & 0 \\ \frac{I_1^{SC}}{I_1^{G1}} & \frac{I_2^{SC}}{I_2^{G2}} & \frac{I_3^{SC}}{I_3^{G3}} & -\mu_4\delta_4^{SC} \end{pmatrix} \cdot \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi 4 \end{pmatrix} = [C]^{-1} \cdot [\xi]. \tag{9}$$

From here, we find the currents:

$$I_1 = \frac{\rho I_1}{\rho 1} = \frac{I_1^{SC} \cdot m_1}{\frac{I_1^{SC}}{I_1^{G1}} m_1 + \frac{I_2^{SC}}{I_2^{G2}} m_2 + \frac{I_3^{SC}}{I_3^{G3}} m_3 - \mu_4 \delta_4^{SC}}, \quad I_2 = \frac{\rho I_2}{\rho 1}, \quad I_3 = \frac{\rho I_3}{\rho 1}.$$

Then, the normalized expressions for currents are

$$\frac{I_1}{I_1^{G1}} = \frac{\frac{I_1^{SC}}{I_1^{G1}} \cdot m_1}{\frac{I_1^{SC}}{I_1^{G1}} \cdot (m_1 - 1) + \frac{I_2^{SC}}{I_2^{G2}} \cdot (m_2 - 1) + \frac{I_3^{SC}}{I_3^{G3}} (m_3 - 1) + 1} = \frac{\frac{I_1^{SC}}{I_1^{G1}} \cdot m_1}{(m_1, m_2, m_3)}$$

$$\frac{I_2}{I_2^{G2}} = \frac{\frac{I_2^{SC}}{I_2^{G2}} \cdot m_2}{(m_1, m_2, m_3)}, \quad \frac{I_3}{I_3^{G3}} = \frac{\frac{I_3^{SC}}{I_3^{G3}} \cdot m_3}{(m_1, m_2, m_3)} \tag{10}$$

The system of equations (10) allows obtaining in a relative form the equations of load characteristics which correspond to the system of equations (1). For this purpose, we express the non-uniform coordinates m_1, m_2, m_3 by the currents and voltages:

$$m_1 = \frac{Y_{L1}}{Y_{L1} - Y_{L1}^{G1}} = \frac{I_1 / I_1^{G1}}{(I_1 / I_1^{G1}) - V_1 / V_1^{G1}}, \; m_2 = \frac{I_2 / I_2^{G2}}{(I_2 / I_2^{G2}) - V_2 / V_2^{G2}}, \; m_3 = \frac{I_3 / I_3^{G3}}{(I_3 / I_3^{G3}) - V_3 / V_3^{G3}}.$$

Having substituted these values in system (10), we obtain the required equations:

$$\begin{aligned} \frac{I_1}{I_1^{G1}} &= \frac{V_1}{V_1^{G1}}\left(1 - \frac{I_1^{SC}}{I_1^{G1}}\right) - \frac{I_1^{SC}}{I_1^{G1}}\frac{V_2}{V_2^{G2}} - \frac{I_1^{SC}}{I_1^{G1}}\frac{V_3}{V_3^{G3}} + \frac{I_1^{SC}}{I_1^{G1}}, \\ \frac{I_2}{I_2^{G2}} &= -\frac{I_2^{SC}}{I_2^{G2}}\frac{V_1}{V_1^{G1}} + \frac{V_2}{V_2^{G2}}\left(1 - \frac{I_2^{SC}}{I_2^{G2}}\right) - \frac{I_2^{SC}}{I_2^{G2}}\frac{V_3}{V_3^{G3}} + \frac{I_2^{SC}}{I_2^{G2}} \\ \frac{I_3}{I_3^{G3}} &= -\frac{I_3^{SC}}{I_3^{G3}}\frac{V_1}{V_1^{G1}} - \frac{I_3^{Sc}}{I_3^{G3}}\frac{V_2}{V_2^{G2}} + \frac{V_3}{V_3^{G3}}\left(1 - \frac{I_3^{SC}}{I_3^{G3}}\right) + \frac{I_3^{SC}}{I_3^{G3}} \end{aligned} \tag{11}$$

The obtained equations represent the purely relative expressions. The currents I_1^{G1}, I_2^{G2}, I_3^{G3} represent scales. It is obvious that relative expressions (11) are generalized to any number of the loads in a formal way. If we switch to absolute values of the regime parameters of system (11), then Y-parameters are expressed through the parameters of the characteristic regimes.

Relevant example. The active network in Fig. 1 is described by the following system:

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -1.236 & 0.17 & 0.282 \\ 0.17 & -0.966 & 0.188 \\ 0.282 & 0.188 & 1.283 \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} + \begin{pmatrix} 2.641 \\ 1.761 \\ 2.817 \end{pmatrix}$$

Hereinafter, the dimensions of values are not specified for simplifying the record and source $V_0 = 5$.

The parameters of the centers of bunch are $I_1^{G1} = 15, Y_{L1}^{G1} = -1.5$, $I_2^{G2} = 16.25$, $Y_{L2}^{G2} = -1.0833$, $I_3^{G3} = 14.844, Y_{L3}^{G3} = -1.583$. Let the actual parameters of the running regime be $Y_{L1}^1 = 0.5$, $Y_{L2}^1 = 0.5, Y_{L3}^1 = 1$, $I_1^1 = 0.974$, $I_2^1 = 0.82, I_3^1 = 1.61$; the nonuniform projective coordinates $m_1^1 = 0.25$, $m_2^1 = 0.316$, $m_3^1 = 0.387$; the distances $\mu_4 \delta_4^{SC} = -0.526$, $\mu_3 \delta_3^1 = -0.776$, $\mu_3 = 0.0907$; the homogeneous coordinates $\rho \xi_1^1 = 0.974/2.641 = 0.369$, $\rho \xi_2^1 = 0.466$, $\rho \xi_3^1 = 0.571$, $\rho \xi_4^1 = 1.476$.

The matrix of transformation (9):

$$[C]^{-1} = \begin{pmatrix} 2.641 & 0 & 0 & 0 \\ 0 & 1.761 & 0 & 0 \\ 0 & 0 & 2.817 & 0 \\ 0.176 & 0.108 & 0.19 & 0.526 \end{pmatrix}$$

3. Comparison of operating regimes and parameters of active two-ports

Let us take two active two-ports with different parameters. The bunches of the load characteristics and conformity of points of characteristic and running regimes are presented in Fig. 4.

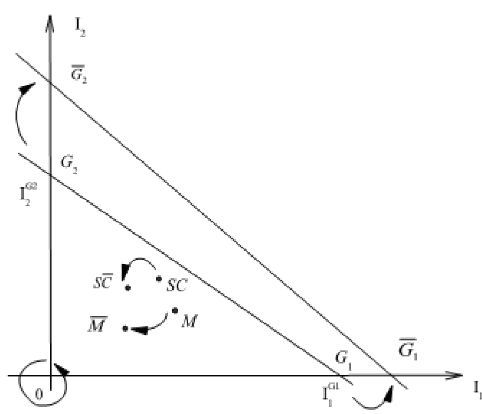

Fig. 4. Conformity of the points of characteristic and running regimes of two active two-ports with various parameters

The problem is to find such values of conductivities and currents of loads of the second two-port (point $\overline{M}$) that its relative regimes would be equal to regimes of the first two-port (point M with preset values of conductivities Y_{L1}, Y_{L2} and currents I_1, I_2). The recalculation of the load conductivities $\overline{Y}_{L1}$, $\overline{Y}_{L2}$ is carried out according to (6):

$$\frac{Y_{L1}}{Y_{L1}-Y_{L1}^{G1}}=\frac{\overline{Y}_{L1}}{\overline{Y}_{L1}-\overline{Y}_{L1}^{G1}},\ \frac{Y_{L2}}{Y_{L2}-Y_{L2}^{G2}}=\frac{\overline{Y}_{L2}}{\overline{Y}_{L2}-\overline{Y}_{L2}^{G2}} \tag{12}$$

Therefore, it is necessary to find a transformation which provides a recalculation of currents. For this purpose, we consider the systems of projective coordinates of these two-ports. Systems of coordinates are set by triangles of reference $G_1\ 0\ G_2$, $\overline{G}_1\ 0\ \overline{G}_2$ and unit points SC, $S\overline{C}$, respectively. Conditions of equality of regimes lead to the equality of projective coordinates $[\overline{\rho}\overline{\xi}]=[\xi]$ of points M and $\overline{M}$. According to (8), for these two-ports $[\rho\xi]=[C]\cdot[I]$, $[\overline{\rho}\overline{\xi}]=[\overline{C}]\cdot[\overline{I}]$. Therefore, a necessary transformation is the product $[\overline{\rho}\overline{I}]=[\overline{C}]^{-1}\cdot[C]\cdot[I]=[J]\cdot[I]$ of two matrixes.

Let us present in detail this product:

$$[J]=\begin{pmatrix}\overline{I}_1^{SC} & 0 & 0\\ 0 & \overline{I}_2^{SC} & 0\\ \dfrac{\overline{I}_1^{SC}}{\overline{I}_1^{G1}} & \dfrac{\overline{I}_2^{SC}}{\overline{I}_2^{G2}} & -\overline{\mu}_3\overline{\delta}_3^{SC}\end{pmatrix}\cdot\begin{pmatrix}\dfrac{1}{I_1^{SC}} & 0 & 0\\ 0 & \dfrac{1}{I_2^{SC}} & 0\\ \dfrac{1}{I_1^{G1}\mu_3\delta_3^{SC}} & \dfrac{1}{I_2^{G2}\mu_3\delta_3^{SC}} & -\dfrac{1}{\mu_3\delta_3^{SC}}\end{pmatrix}$$

$$=\begin{pmatrix}J_{11} & 0 & 0\\ 0 & J_{22} & 0\\ J_{31} & J_{32} & J_{33}\end{pmatrix}$$

In turn, the matrix elements are

$$J_{11}=\frac{\overline{I}_1^{SC}}{I_1^{SC}},\ J_{22}=\frac{\overline{I}_2^{SC}}{I_2^{SC}},\ J_{33}=\frac{\overline{\mu}_3\overline{\delta}_3^{SC}}{\mu_3\delta_3^{SC}},$$

$$J_{31}=\frac{\overline{\mu}_3\overline{\delta}_3^{SC}}{I_1^{G1}\mu_3\delta_3^{SC}}\left(\frac{\overline{I}_1^{SC}I_1^{G1}}{I_1^{SC}\overline{I}_1^{G1}}\cdot\frac{\mu_3\delta_3^{SC}}{\overline{\mu}_3\overline{\delta}_3^{SC}}-1\right),\ J_{32}=\frac{\overline{\mu}_3\overline{\delta}_3^{SC}}{I_2^{G2}\mu_3\delta_3^{SC}}\left(\frac{\overline{I}_2^{SC}I_2^{G2}}{I_2^{SC}\overline{I}_2^{G2}}\cdot\frac{\mu_3\delta_3^{SC}}{\overline{\mu}_3\overline{\delta}_3^{SC}}-1\right)$$

The form of these expressions shows that we can divide all the elements by J_{33} and introduce the values:

$$\overline{m}_1=\frac{\overline{I}_1^{SC}}{I_1^{SC}}\frac{\mu_3\delta_3^{SC}}{\overline{\mu}_3\overline{\delta}_3^{SC}},\ \overline{m}_2=\frac{\overline{I}_2^{SC}}{I_2^{SC}}\frac{\mu_3\delta_3^{SC}}{\overline{\mu}_3\overline{\delta}_3^{SC}}$$

Then, the required transformation becomes

$$\begin{pmatrix}\overline{\rho}\,\overline{I}_1\\ \overline{\rho}\,\overline{I}_2\\ \overline{\rho}1\end{pmatrix}=\begin{pmatrix}\overline{m}_1 & 0 & 0\\ 0 & \overline{m}_2 & 0\\ \dfrac{1}{I_1^{G1}}\left(\overline{m}_1\dfrac{I_1^{G1}}{\overline{I}_1^{G1}}-1\right) & \dfrac{1}{I_2^{G2}}\left(\overline{m}_2\dfrac{I_2^{G2}}{\overline{I}_2^{G2}}-1\right) & 1\end{pmatrix}\cdot\begin{pmatrix}I_1\\ I_2\\ 1\end{pmatrix} \tag{13}$$

From here, we express the currents:

$$\bar{I}_1 = \frac{\overline{m}_1 \cdot I_1}{\frac{I_1}{I_1^{G1}}\left(\overline{m}_1 \frac{I_1^{G1}}{\bar{I}_1^{G1}} - 1\right) + \frac{I_2}{I_2^{G2}}\left(\overline{m}_2 \frac{I_2^{G2}}{\bar{I}_2^{G2}} - 1\right) + 1}, \quad \bar{I}_2 = \frac{\overline{m}_2 \cdot I_2}{\frac{I_1}{I_1^{G1}}\left(\overline{m}_1 \frac{I_1^{G1}}{\bar{I}_1^{G1}} - 1\right) + \frac{I_2}{I_2^{G2}}\left(\overline{m}_2 \frac{I_2^{G2}}{\bar{I}_2^{G2}} - 1\right) + 1} \quad (14)$$

The structure of expressions (14) shows that it is possible to introduce the normalized values:

$$\overline{m}_1^N = \overline{m}_1 \frac{I_1^{G1}}{\bar{I}_1^{G1}}, \quad \overline{m}_2^N = \overline{m}_2 \frac{I_2^{G2}}{\bar{I}_2^{G2}}.$$

Then, we obtain the normalized form of transformation (14):

$$\begin{pmatrix} \bar{\rho} \frac{\bar{I}_1}{\bar{I}_1^{G1}} \\ \bar{\rho} \frac{\bar{I}_2}{\bar{I}_2^{G2}} \\ \bar{\rho} 1 \end{pmatrix} = \begin{pmatrix} \overline{m}_1^N & 0 & 0 \\ 0 & \overline{m}_2^N & 0 \\ \overline{m}_1^N - 1 & \overline{m}_2^N - 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{I_1}{I_1^{G1}} \\ \frac{I_2}{I_2^{G2}} \\ 1 \end{pmatrix}$$

These transformations allow carrying out the recalculation of currents of the second two-port for any values of currents of the first two-port using the pre-calculated values of $\overline{m}_1, \overline{m}_2$ or $\overline{m}_1^N, \overline{m}_2^N$.

4. Conclusion

1. Projective geometry adequately interprets "kinematics" of a circuit with changeable parameters of elements and yields the relationships useful in practice. The given approach is applicable to the analysis of «flowed » form processes of different physical nature.
2. The derived equations of the active multiport network in the normalized form define the scales for the currents and conductivities and allow comparing regimes of different circuits.
3. From the methodological point of view, the presented approach is applied in other scientific domains, such as mechanics (the principles of special relativity) and biology (the principles of age changes of plants and organisms [6]).

References

[1] L.A. Bessonov, Teoreticheskie osnovy elektrotekhniki: Elektricheskie tsepi. Vysshaya Shkola, 1996.

[2] A.A. Penin, International Journal of Electrical and Computer Engineering. 3. 15, 968, (2008). http://www.waset.org/journals/ijece/v3/v3-15-146.pdf

[3] A.A.Penin, Wseas Transactions on Circuits and Systems. 10, 5, 157, (2011). http://www.wseas.us/e-library/transactions/circuits/2011/53-346.pdf.

[4] N.A.Glagolev, Proektivnaya geometriya, Nauka, 1963.

[5] R. Casse, Projective geometry: An introduction. New York, 2006.

[6] S.V.Petukhov, Geometrii zhivoi prirody i algoritmy samoorganizatsii. Novoe v zhizni, nauke, tehnike, Seriya “Matematika, kibernetika”, Znanie, 6, 1988.